

PHY7400. Homework 1

This homework assignment is due on **September 19**.

Suggested reading:

Eugen Merzbacher, *Quantum Mechanics*, Chapters 2-4.

Problem 1: Fun with operators.

In class, we discussed the properties of operators used in quantum mechanics. Let us prove some of the identities mentioned there.

1. Show that *any* linear operator \hat{L} can be represented as $\hat{L} = \hat{A} + i\hat{B}$, where \hat{A} and \hat{B} are Hermitian operators. **(2 pt)**
2. Consider the commutators $[\hat{A}, \hat{B}\hat{C}]$ and $[\hat{A}\hat{B}, \hat{C}]$ of any three linear operators \hat{A} , \hat{B} , and \hat{C} . Show that they can be expressed via the commutators $[\hat{A}, \hat{B}]$, $[\hat{A}, \hat{C}]$, and $[\hat{B}, \hat{C}]$. **(3 pt)**
3. Consider the following operator, $(\hat{A} - \lambda\hat{B})^{-1}$. Assuming that λ is small, expand this operator in a power series in λ . Hint: write $(\hat{A} - \lambda\hat{B})^{-1} = \sum_n \lambda^n \hat{C}_n$ and determine \hat{C}_n . **(5 pt)**

Problem 2: More fun: functions of operators.

An operator $\hat{F} = F(\hat{f})$, where $F(z)$ is a function of z which can be expanded as $F(z) = \sum_n c_n z^n$, can be understood as an operator $\hat{F} = \sum_n c_n \hat{f}^n$. Using this definition and resumming the resulting series, find an explicit form of the following operators

1. $\hat{R}_a = \exp(ia\hat{I})$, where $\hat{I}\psi(x) = \psi(-x)$ is a reflection operator, **(2 pt)**
2. Displacement operator $\hat{T}_a = \exp\left(a\frac{d}{dx}\right)$, **(4 pt)**
3. Rescaling operator $\hat{L}_a = \exp\left(ax\frac{d}{dx}\right)$, **(4 pt)**

where a is a real parameter.

Problem 3: Even more fun: Jacobi identity.

Show that any three linear operators \hat{A} , \hat{B} , and \hat{C} satisfy the following (Jacobi) identity (**10 pt**)

$$[[\hat{A}, \hat{B}], \hat{C}] + [[\hat{B}, \hat{C}], \hat{A}] + [[\hat{C}, \hat{A}], \hat{B}] = 0 \quad (1)$$