PHY7400. Homework 5

This homework assignment is due on **November 4**.

Suggested reading:

Eugen Merzbacher, *Quantum Mechanics*, Chapters 6-7
David J. Griffiths, *Introduction to Quantum Mechanics*, Chapter 2.

Problem 1: Fun with the square well (10 pt).

Find energy spectrum of a particle confined in the potential

$$U(x) = \begin{cases} \alpha\delta(x), & \text{for } |x| < a \\ \infty, & \text{for } |x| \geq a, \end{cases} \quad (1)$$

where $\alpha > 0$. How does delta-function potential affect the energy levels in this problem compared to the case of ordinary square well potential discussed in class? In addition

1. show that if $m\alpha a/\hbar^2 \gg 1$ then the low energy spectrum consists of pairs of closely-spaced (in energy) levels
2. find the equation whose solutions describe energy levels in the potential

$$U(x) = \begin{cases} \alpha\delta(x-b), & \text{for } |x| < a \\ \infty, & \text{for } |x| \geq a, \end{cases} \quad (2)$$

   for $b < a$ and show that it reduces to the original equation for $b = 0$

Problem 2: Potential step revisited (10 pt).

Consider potential described by a continuous real function $U(x)$ such that

$$U(x) = \begin{cases} 0, & \text{for } x \to -\infty \\ U_0, & \text{for } x \to \infty, \end{cases} \quad (3)$$

where $U_0 > 0$. Show that the transmission coefficient $T(E) \sim \sqrt{E - U_0} \to 0$ for $E \to U_0$. Compare to the case of potential step discussed in class.

Problem 3: Fun with the ladder operators (10 pt).

Consider differently normalized ladder operators $\hat{a}$ for particle in a harmonic oscillator potential discussed in class:

$$\hat{a}^\dagger = \frac{1}{\sqrt{2}}(\hat{Q} - i\hat{P}), \hat{a} = \frac{1}{\sqrt{2}}(\hat{Q} + i\hat{P}), \tag{4}$$

where $\hat{Q} = x(\sqrt{m\omega/\hbar})$ and $\hat{P} = \hat{p}(1/\sqrt{\hbar m\omega})$.

1. Write the harmonic oscillator Hamiltonian in terms of $\hat{P}$ and $\hat{Q}$ and in terms of $\hat{a}^\dagger$ and $\hat{a}$.
2. Show that for harmonic oscillator eigenfunctions $\psi_n(x)$

$$\hat{a}_+\psi_n(x) = \sqrt{n+1}\psi_{n+1}(x) \tag{5}$$

3. Finally, find all non-zero matrix elements of operators $\hat{P}$ and $\hat{Q}$ between different harmonic oscillator wavefunctions $\psi_n(x)$ and $\psi_k(x)$. Hint: express $\hat{P}$ and $\hat{Q}$ in terms of $\hat{a}$ and $\hat{a}^\dagger$.