Advanced Topics in EFT

Homework 1 Due: Monday, October 6, 5PM

1. (a) Compute the dimensions of the following representations in $SU(N)$:

(b) Compute the following products of representations in $SU(5)$:

Express your final answers in terms of the dimensions, being sure to include whether or not it’s the conjugate representation, if necessary.

2. In class we showed that in $SU(3)$: $\mathbf{8} \otimes \mathbf{3} = \mathbf{15} \oplus \mathbf{6} \oplus \bar{\mathbf{3}}$. Prove this explicitly by symmetrizing indices. Hint: Recall that invariant tensors are either symmetric and traceless, or antisymmetric. Be sure to keep track of covariant vs contravariant indices – it will make your life much easier!

3. Consider the quantum D=3 simple harmonic oscillator.

(a) When written in terms of creation/annihilation operators, it is immediately clear that the system has an $SU(3)$ symmetry. Find the conserved charges of this symmetry.

(b) Since this is a symmetry of the Hamiltonian, we know that there will be degeneracies among the energy eigenstates. Find these degeneracies and show that they completely explain the degeneracies that you calculated in your quantum class! Classify the energy eigenstates in terms of $SU(3)$ representations.

(c) Show that built into this symmetry, there lives an $SU(2)$ subgroup that can be identified with the overall rotational symmetry of the problem. Use this to derive the angular momentum operator $L$.

(d) Suppose I add a term to the Hamiltonian $\Delta H = \alpha L^2$. Describe what happens to the first four energy levels.