1. The Cartan Algebra of the group $SU(n + 1)$ in the fundamental representation is spanned by the traceless matrices

$$H_k = \frac{1}{\sqrt{2k(k+1)}} \text{Diag}\{1,\ldots,1,-k,0,\ldots,0\} \quad k = 1,\ldots,n$$

where there are $k$ “1”s and we’re following the usual convention that $\text{Tr}[T^a T^b] = \frac{1}{2} \delta^{ab}$.

The other generators are various embeddings of the Pauli matrices; see your favorite Group theory text for details. Consider the symmetry breaking $SU(4) \rightarrow SU(3) \times U(1)$.

(a) Consider how a field in the fundamental of $SU(4)$ transforms under the symmetry after breaking. Find both its representation under $SU(3)$ and its charge under the $U(1)$. Hint: A fundamental of $SU(N)$ transforms as $\delta^a F^i = i[T^a]^j_i F^j$.

(b) Repeat the above for a field in the adjoint of $SU(4)$. Hint: An adjoint of $SU(N)$ transforms as $\delta^a A^b = i[T^a, A^b]$.

(c) Assume that there is a term in the Hamiltonian of the system that transforms in the fundamental (4) representation. Using the generalization of the Wigner-Eckart Theorem to $SU(N)$, how many reduced matrix elements are required to describe the transitions $(1,0,0) \rightarrow (3,0,0), (1,1,0)$?

2. Using arguments of effective field theory, explain why the sky is blue:

(a) Identify the relevant degrees of freedom and energy/length scales.

(b) Write down the free action and construct a scaling rule for each relevant field.

(c) Find the leading operators and use them to “compute” the scattering cross section of light in the atmosphere.

Note: this is not a Jackson problem – do not treat it as such!!

3. Finish what we started in class by embedding QED into the $SU(3)$ Chiral Lagrangian. Find the currents in terms of the $\Sigma$ field.