Advanced Topics in EFT

Homework 3 Solutions

Problem 1:
Reading assignment.

Problem 2:

We have an $SU(N)$ gauge theory, with $N \geq 3$, so we must make sure that that theory has no gauge anomalies; it may or may not have global anomalies. With $L$ fermions in the $\Box$ and $R$ fermions in the $\Box$ the anomaly coefficients are easy to calculate:

$$A = L(N + 4) + R(-1) = 0 \Rightarrow R = (N + 4)L$$

All other anomaly coefficients vanish, thanks to the traceless conditions on the generators of $SU(M)$, for $M = N, L, R$.

Problem 3:

We have the following table:

<table>
<thead>
<tr>
<th>$\psi_L$</th>
<th>$SU(N)$</th>
<th>$SU(F)_L$</th>
<th>$SU(F)_R$</th>
<th>$U(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_R$</td>
<td>$\Box$</td>
<td>1</td>
<td>$Q_L$</td>
<td>\n</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$\Box$</td>
<td>1</td>
<td>$Q_R$</td>
<td>\n</td>
</tr>
<tr>
<td></td>
<td>$\Box$</td>
<td>1</td>
<td>$Q_A$</td>
<td>\n</td>
</tr>
</tbody>
</table>

1. In the first step we ignore the $\xi$ field. Notice first that the $SU(N)^3$ gauge anomaly vanishes since the theory has vectorlike pair of fermions. There is a potential nonvanishing anomaly:

$$A_{NNQ} = \frac{F}{2}(Q_L + Q_R)$$

This implies that $Q_L = -Q_R$ to avoid this anomaly. The global anomalies $A_{QQQ}, A_Q$ (the latter being the gravitational anomaly) automatically vanish now. There is still the $SU(F)$ global anomalies:

$$A_{LLQ} = \frac{N}{2}Q_L$$
$$A_{RRQ} = \frac{N}{2}Q_R$$
These only vanish if $Q_L = Q_R = 0$. If we accept the presence of this global anomaly (and why shouldn’t we?!) then this constraint is lifted and there can be a $U(1)$, so long as it’s vectorlike.

Notice also that the sum of the $L$ and $R$ fermions, with the charges equal and opposite, has a vanishing anomaly, while the difference has a nonvanishing anomaly. That is: the vector isospin current is anomaly free, while the axial isospin current is not, exactly as we expect.

2. Now we include the adjoint $\xi$. Again, the pure gauge anomaly vanishes, since the fermions are in a real representation. We have three nontrivial anomalies (ignoring the $SU(F)$ anomalies):

$$A_{NNQ} = \frac{F}{2}(Q_L + Q_R) + NQ_A$$
$$A_{QQQ} = NF(Q_L^3 + Q_R^3) + (N^2 - 1)Q_A^3$$
$$A_Q = NF(Q_L + Q_R) + (N^2 - 1)Q_A$$

The first and third of these anomaly coefficients vanishing leads to the condition $Q_A = 0$. Then the second anomaly $A_{QQQ}$ leads to the same constraint as before. This should not surprise us too much: after all, the $\text{Adj}$ is a real representation, so it should not be able to develop a phase.