Spontaneous Symmetry Breaking and the Higgs Mechanism

Andrew E. Blechman

December 13, 2000

Abstract

This is a writeup of a lecture given in PHY 254 (Fall, 2000) on Spontaneous Symmetry Breaking and the Higgs Mechanism. A simple potential is presented and the symmetry is broken. A comparison with Quantum Field Theory allows us to see how this introduces a massive and a massless scalar (spin-0) boson. The Higgs Mechanism is introduced and we see how the intermediate vector bosons gain mass. The principle of gauge invariance is used to remove the Goldstone boson and add a degree of freedom to the polarization state of the now massive vector bosons, leaving us with the massive Higgs and vector bosons.

1 Introduction

So far, we have been considering the $W^{\pm}$ and Z^0 vector bosons as the force carriers of the weak force. In doing this, we make them act the same way as photons (with different couplings), and the propogator becomes:

$$-\frac{g^{\mu\nu}-\frac{q^{\mu}q^{\nu}}{m^2}}{q^2-m^2}$$

In this sense, the $W^{\pm}$ and Z^0 vector bosons are exactly analogous to the photon. The theory that treats these bosons this way is oftentimes called the theory of "Intermediate Vector Bosons" (IVB). However, this theory is fundamentally flawed, because one can show that it is not renormalizeable, and therefore unsuited to describe any higher-order processes. We therefore need a new way to describe the theory of electroweak interactions.

The central mathematical trick behind this new theory is the idea of "spontaneous symmetry breaking" (SSB). In this paper, we present an example of this in the context of Quantum Field Theory (QFT); then we present the Higgs Mechanism, where SSB solves our problem and leaves us with a gauge-invariant (and therefore renormalizeable) theory of electroweak interactions.

2 Spontaneous Symmetry Breaking

Consider the following potential:

$$V(x,y) = -\frac{1}{2}\mu^2(x^2+y^2) + \frac{1}{4}\lambda^2(x^2+y^2)^2 \tag{1}$$

This potential is very similar to an anharmonic oscillator, except for the negative sign of the squared term. It has a point of symmetry at the origin of the xy-plane. We can easily calculate all the extrema of this potential:

$$\frac{\partial V}{\partial x} = -x(\mu^2 - \lambda^2(x^2 + y^2)) = 0 \tag{2}$$

$$\frac{\partial V}{\partial y} = -y(\mu^2 - \lambda^2(x^2 + y^2)) = 0 \tag{3}$$

$$x = y = 0 \tag{4}$$

$$x^2 + y^2 = \frac{\mu^2}{\lambda^2} \tag{5}$$

One notices right away that our symmetry point is an unstable point of equilibrium (e.g.: $\frac{\partial^2 V}{\partial x^2}(0,0) \leq 0$). It might therefore be wiser to choose another point as our "vacuum". For the purposes of this paper, we will choose:

$$x_0 = 0 \tag{6}$$

$$y_0 = \frac{\mu}{\lambda} \tag{7}$$

It is important to notice right away that this is not the only point we could have chosen; we could have chosen any point that lies on the circle in Equantion 5. However, once we choose a point, we must be consistant and stick to that point.

Now we can define a new coordinate system that allows for perturbations about our vacuum (x_0, y_0):

$$\xi = x \tag{8}$$

$$\eta = y - \frac{\mu}{\lambda} \tag{9}$$

This has the effect of moving our origin to our vaccum point. Under this change of coordinates, our potential becomes:

$$V(x \to \xi, y \to \eta) = -\frac{1}{2}\mu^2(\xi^2 + (\eta + \frac{\mu}{\lambda})^2) + \frac{1}{4}\lambda^2(\xi^2 + (\eta + \frac{\mu}{\lambda})^2)^2 \tag{10}$$

$$= \mu^2\eta^2 + \mathcal{O}(3^{rd} - \text{order terms}) \tag{11}$$

where we have dropped any constants since they do not affect the potential difference. Further, we have grouped all higher order terms and set them aside. These terms are the terms that define couplings for Feynman Diagrams; they are irrelevant to this discussion.

Now we can consider how this fits into a QFT picture. Recall in Classical Mechanics how we can define a Lagrangian for a system, usually by the formula L = T - V. It turns out that we can do the same thing for a field. [1] We define a functional $\mathcal{L}$ such that:

$$\mathcal{L} = \mathcal{T} - \mathcal{V} \tag{12}$$

$$L = \int \mathcal{L}\ d^3x \tag{13}$$

By plugging this Lagrangian into the Euler-Lagrange equations of Classical Dynamics, we can rederive the field equations. $\mathcal{L}$ is sometimes called a *Lagrangian Density*, but we shall simply call it a Lagrangian. Also notice that Equation 12 is not necessarely an accurate way of describing the Lagrangian, but it allows us to make the analogy with our intuition. By $\mathcal{T}$, we mean terms that involve derivatives of the fields. We are not concerned with these terms in our discussion; we will only be concerned with the terms in $\mathcal{V}$.

Now we draw attention to a special kind of field in relativistic quantum mechanics known as a *Klein-Gordon field.* This term has a potential term that has the form:

$$\mathcal{V}_{KG} = m^2\phi^2 \tag{14}$$

where m is the "mass" of the field, and ϕ is the field itself. ϕ is a scalar, so it must correspond to a particle of spin-0.

Now let us compare Equation 14 to Equation 11. We notice from direct comparison that we have two fields:

1. η corresponds to a field (particle) of mass μ and spin-0 $\Rightarrow$ Higgs Boson

2. ξ corresponds to a field (particle) of mass 0 and spin-0 $\Rightarrow$ Goldstone Boson

This introduction of a massive and massless scalar field in SSB is a very well-known result proved by Goldstone (known as the Goldstone Theorem). More generally, it says that whenever you spontaneously break the symmetry of a system, you introduce *at least* one massless scalar field. Unfortunately, we have never seen these bosons before, so they cannot exist in nature (since they are massless, if they did exist, we should have seen them). We must find a way to remove them from our theory if SSB is to work.

[1] This is true for classical as well as quantum fields; cf: Maxwell's Lagrangian, or Einstein's Lagrangian in Jackson, *Classical Electrodynamics* or Landau and Lifshitz, *Classical Theory of Fields*.

3 Higgs Mechanism

In 1964, Peter Higgs and others (including Rochester's own C.R. Hagen) were working to try and figure out how and why the intermediate vector bosons have mass. Quantum field theorists have known for a long time how to introduce a vector field into the Lagrangian: to make the theory covariant, you introduce the covariant derivative which gives you a new vector field (A^μ). Compare this new vector field with the Vector-Field Lagrangian (called the *Proca Lagrangian*):

$$\mathcal{L}_{Proca} = \mathcal{T} + \mu^2 A^2 + \mathcal{O}(3^{rd} - \text{order terms}) \tag{15}$$

One can properly introduce the vector field and keep the gauge invariance, *as long as the vector field is massless!* Once you introduce massive vector fields, the theory looses its gauge invariance, and by 'tHooft and Veltmann's marvelous theorem, the theory is no longer renormalizable. These problems could be avoided by introducing SSB into the theory, but no one knew how to get rid of the Goldstone boson.

After breaking the symmetry, we notice that by simply manipulating terms and comparing with the Proca Lagrangian, we end up with a mass term for the vector bosons, while preserving gauge invariance. This new term goes as (in natural units):

$$m_A \rightarrow \frac{e\mu}{\lambda} \tag{16}$$

This is not hard to see, but involves using gauge invariance of the Lagrangian and manipulating terms. [2] However, even though the vector fields have mass, we are still left with a massless scalar field that we know does not exist, namely the Goldstone boson. What do we do with that?

The answer: we make it go away! Recall how our field was originally parametrized in terms of x and y (see Equation 1). We will now reparametrize it using complex notation:

$$\phi \equiv x + iy \qquad\qquad (|\phi|^2 = x^2 + y^2) \tag{17}$$

$$V(x,y) \rightarrow V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4 \tag{18}$$

In this notation, we notice that the mass term makes sense: this almost looks like an oscillator with a mode of μ/λ. Now we must invoke gauge invariance: Consider the general field transformation:

$$\phi \rightarrow \phi' \equiv e^{i\theta(x,y)} \Rightarrow \mathcal{L}(\phi) \rightarrow \mathcal{L}' \equiv \mathcal{L}(\phi') \tag{19}$$

[2] For details on how this is done, we refer you to Griffiths, *Introduction to Elementary Particles*, Ch 11.

$$\text{Then}: \ \mathcal{L}' = \mathcal{L} \tag{20}$$

This is the principle of gauge invariance. Let's apply this general principle to our field:

$$\phi \to \phi e^{i\theta} = (x+iy)(\cos\theta + i\sin\theta) = (x\cos\theta - y\sin\theta) + i(x\sin\theta + y\cos\theta) \tag{21}$$

$$\text{Let} \ \ \theta \equiv \tan^{-1}(x/y) \tag{22}$$

With this choice of θ, we notice that ϕ is pure imaginary, which means that x $\Leftrightarrow \xi = 0$. Therefore, the Goldstone boson has vanished!

What is really going on here? It can't be that we have a boson in one gauge and nothing in another; that would simply be inconsistant. To answer this question, let us look at the big picture. We started off with massless vector fields. We know from electrodynamics that such fields can only be polarized in two directions (transverse): the longitudinal polarization does not exist. Now we have given the particles mass, and therefore they should gain a third (longitudinal) polarization state. Where did this extra degree of freedom come from? *It came from the Goldstone Boson!* We removed the Goldstone boson from the theory in favor of a third polarization state for the now massive vector fields. To quote Griffiths: "The [vector] fields 'ate' the Goldstone boson, thereby acquiring both a mass and a third polarization state." We actually could have eliminated the Goldstone boson from the beginning by introducing a third polarization state for the massless vector fields, but this introduction of "ghost particles" is not standard.

Peter Higgs was the first person to see how this method can be used to give the vector fields mass and remove the Goldstone boson, leaving us with only one massive boson yet to be discovered. For this reason, it is called the "Higgs Mechanism", and the massive boson is called the "Higgs boson".

4 Conclusions

This lecture was an attempt to explain the intricate beauty of Spontaneous Symmetry Breaking and the Higgs Mechanism without getting into the details of formal quantum field theory. It was a challenging and arduous task! It is very difficult to truly appreciate what is going on unless you can actually do the manipulation of the Lagrangian and invoke gauge invariance. However, even with this watered-down version of the mechanisms involved, we can see several important points. One point that was not brought up in the lecture was the final result that now we have an gauge-invariant theory, and therefore we know that it is renormalizable, thanks to 'tHooft and Veltmann. The proof of this is remarkably subtle and complex, and actually involves putting the Goldstone boson back into the theory (the so-called "'tHooft gauge"). We do not discuss the details here, but one can certainly begin to appreciate all the doors that the Higgs mechanism opened for High Energy Physics.