Chapter 26

Special Relativity

Lecture 13

Review problem: Gallilean Relativity
Airplanes in Flight

Two airplanes fly paths I and II as specified in figure below. Both planes have airspeeds of 100 m/s and fly a distance \( L = 200 \text{ km} \). The wind blows at 20.0 m/s in the direction shown in the figure. Find (a) the time of flight to each city, (b) the time to return.

Given:
- \( L = 200 \text{ km} \)
- \( v_p = 100 \text{ m/s} \)
- \( v_w = 20.0 \text{ m/s} \)

Find:
- \( t = ? \)

Recall that the ground speed of the plane is

\[ v_{gr} = v_{air} + v_{wind} \]

First plane (path OB):

\[ v_{p_{OB}} = 100 \text{ m/s} + 20 \text{ m/s} = 120 \text{ m/s} \]

\[ t_{OB} = \frac{200 \times 10^3 \text{ m}}{120 \text{ m/s}} = 1.67 \times 10^3 \text{ s} \]

Second plane (path OA):

\[ v_{p_{OA}} = \sqrt{v_{air}^2 - v_{wind}^2} = 98.0 \text{ m/s} \]

\[ t_{OA} = \frac{200 \times 10^3 \text{ m}}{98.0 \text{ m/s}} = 2.04 \times 10^3 \text{ s} \]
Basic Problems

- The speed of every particle in the universe always remains less than the speed of light.
- Newtonian Mechanics is a limited theory:
  - It places no upper limit on speed.
  - It is contrary to modern experimental results.
  - Newtonian Mechanics becomes a specialized case of Einstein’s Theory of Special Relativity.
  - When speeds are much less than the speed of light.

Galilean Relativity – Limitations

- Galilean Relativity does not apply to experiments in electricity, magnetism, optics, and other areas.
- Results do not agree with experiments:
  - The observer should measure the speed of the pulse as \( v + c \).
  - Actually measures the speed as \( c \).

Michelson-Morley Equipment

- Used the Michelson Interferometer.
- Arm 2 is aligned along the direction of the earth’s motion through space.
- The interference pattern was observed while the interferometer was rotated through 90°.
- The effect should have been to show small, but measurable, shifts in the fringe pattern.

Michelson-Morley Results

- Measurements failed to show any change in the fringe pattern:
  - No fringe shift of the magnitude required was ever observed.
- Light is now understood to be an electromagnetic wave, which requires no medium for its propagation:
  - The idea of an ether was discarded.
- The laws of electricity and magnetism are the same in all inertial frames.
Einstein’s Principle of Relativity

- Resolves the contradiction between Galilean relativity and the fact that the speed of light is the same for all observers
- Postulates
  - The Principle of Relativity: All the laws of physics are the same in all inertial frames
  - The constancy of the speed of light: the speed of light in a vacuum has the same value in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light

The Principle of Relativity

- This is a sweeping generalization of the principle of Galilean relativity, which refers only to the laws of mechanics
- The results of any kind of experiment performed in a laboratory at rest must be the same as when performed in a laboratory moving at a constant speed past the first one.
- No preferred inertial reference frame exists
- It is impossible to detect absolute motion

The Constancy of the Speed of Light

- Been confirmed experimentally in many ways
  - A direct demonstration involves measuring the speed of photons emitted by particles traveling near the speed of light
  - Confirms the speed of light to five significant figures
- Explains the null result of the Michelson-Morley experiment
- Relative motion is unimportant when measuring the speed of light
  - We must alter our common-sense notions of space and time

Consequences of Special Relativity

- Restricting the discussion to concepts of length, time, and simultaneity
- In relativistic mechanics
  - There is no such thing as absolute length
  - There is no such thing as absolute time
  - Events at different locations that are observed to occur simultaneously in one frame are not observed to be simultaneous in another frame moving uniformly past the first
Simultaneity

- In Special Relativity, Einstein abandoned the assumption of simultaneity
- Thought experiment to show this
  - A boxcar moves with uniform velocity
  - Two lightning bolts strike the ends
  - The lightning bolts leave marks (A' and B') on the car and (A and B) on the ground
  - Two observers are present: O' in the boxcar and O on the ground

Simultaneity – Thought Experiment Set-up

- Observer O is midway between the points of lightning strikes on the ground, A and B
- Observer O’ is midway between the points of lightning strikes on the boxcar, A’ and B’

Simultaneity – Thought Experiment Results

- The light reaches observer O at the same time
  - She concludes the light has traveled at the same speed over equal distances
  - Observer O concludes the lightning bolts occurred simultaneously

Simultaneity – Thought Experiment Results, cont

- By the time the light has reached observer O, observer O’ has moved
- The light from B’ has already moved by the observer, but the light from A’ has not yet reached him
  - The two observers must find that light travels at the same speed
  - Observer O’ concludes the lightning struck the front of the boxcar before it struck the back (they were not simultaneous events)
Simultaneity – Thought Experiment, Summary

- Two events that are simultaneous in one reference frame are in general not simultaneous in a second reference frame moving relative to the first.
- That is, simultaneity is not an absolute concept, but rather one that depends on the state of motion of the observer.
  - In the thought experiment, both observers are correct, because there is no preferred inertial reference frame.

Time Dilation

- A mirror is fixed to the ceiling of a vehicle.
- The vehicle is moving to the right with speed v.
- An observer, O', at rest in this system holds a laser a distance d below the mirror.
- The laser emits a pulse of light directed at the mirror (event 1) and the pulse arrives back after being reflected (event 2).

Time Dilation, Moving Observer

- Observer O' carries a clock.
- She uses it to measure the time between the events ($\Delta t_p$).
  - She observes the events to occur at the same place.
  - $\Delta t_p = \text{distance}/\text{speed} = (2d)/c$

Time Dilation, Stationary Observer

- Observer O is a stationary observer on the earth.
- He observes the mirror and O' to move with speed v.
- By the time the light from the laser reaches the mirror, the mirror has moved to the right.
- The light must travel farther with respect to O than with respect to O'.
Time Dilation, Observations

- Both observers must measure the speed of the light to be $c$.
- The light travels farther for $O$.
- The time interval, $\Delta t$, for $O$ is longer than the time interval for $O'$, $\Delta t_p$.

Time Dilation, Time Comparisons

- Observer $O$ measures a longer time interval than observer $O'$.
- \[ \Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t_p \]
- Where \[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

Time Dilation, Summary

- The time interval $\Delta t$ between two events measured by an observer moving with respect to a clock is longer than the time interval $\Delta t_p$ between the same two events measured by an observer at rest with respect to the clock.
- A clock moving past an observer at speed $v$ runs more slowly than an identical clock at rest with respect to the observer by a factor of $\gamma^{-1}$.

Identifying Proper Time

- The time interval $\Delta t_p$ is called the proper time.
- The proper time is the time interval between events as measured by an observer who sees the events occur at the same position.
- You must be able to correctly identify the observer who measures the proper time interval.
Alternate Views

- The view of O’ that O is really the one moving with speed v to the left and O’s clock is running more slowly is just as valid as O’s view that O was moving.
- The principle of relativity requires that the views of the two observers in uniform relative motion must be equally valid and capable of being checked experimentally.

Time Dilation – Generalization

- All physical processes slow down relative to a clock when those processes occur in a frame moving with respect to the clock.
  - These processes can be chemical and biological as well as physical.
- Time dilation is a very real phenomena that has been verified by various experiments.

Time Dilation Verification – Muon Decays

- Muons are unstable particles that have the same charge as an electron, but a mass 207 times more than an electron.
- Muons have a half-life of $\Delta t_p = 2.2\mu s$ when measured in a reference frame at rest with respect to them (a).
- Relative to an observer on earth, muons should have a lifetime of $\gamma \Delta t_p$ (b).
- A CERN experiment measured lifetimes in agreement with the predictions of relativity.

\[ v = 0.99c, \gamma = 7.1 \]

QUICK QUIZ 26.1

Imagine that you are an astronaut who is being paid according to the time spent traveling in space as measured by a clock on Earth. You take a long voyage traveling at a speed near that of light. Upon your return to Earth, your paycheck will be:

(a) smaller than if you had remained on Earth, (b) larger than if you had remained on Earth, or (c) the same as if you had remained on Earth.
QUICK QUIZ 26.1 ANSWER

(b). Assuming that your on-duty time was kept on Earth, you will be pleasantly surprised with a large paycheck. Less time will have passed for you in your frame of reference than for your employer back on Earth.

The Twins’ Perspectives

- Goslo’s perspective is that he was at rest while Speedo went on the journey.
- Speedo thinks he was at rest and Goslo and the earth raced away from him on a 6.5 year journey and then headed back toward him for another 6.5 years.
- The paradox – which twin is the traveler and which is really older?

The Twin Paradox – The Situation

- A thought experiment involving a set of twins, Speedo and Goslo.
- Speedo travels to Planet X, 20 light years from earth.
  - His ship travels at 0.95c.
  - After reaching planet X, he immediately returns to earth at the same speed.
- When Speedo returns, he has aged 13 years, but Goslo has aged 42 years.

The Twin Paradox – The Resolution

- Relativity applies to reference frames moving at uniform speeds.
- The trip in this thought experiment is not symmetrical since Speedo must experience a series of accelerations during the journey.
- Therefore, Goslo can apply the time dilation formula with a proper time of 42 years.
  - This gives a time for Speedo of 13 years and this agrees with the earlier result.
- There is no true paradox since Speedo is not in an inertial frame.
Length Contraction

- The measured distance between two points depends on the frame of reference of the observer.
- The proper length, $L_P$, of an object is the length of the object measured by someone at rest relative to the object.
- The length of an object measured in a reference frame that is moving with respect to the object is always less than the proper length.
  - This effect is known as **length contraction**.

Length Contraction – Equation

\[ L = \frac{L_P}{\gamma} = L_P \sqrt{1 - \frac{v^2}{c^2}} \]

- Length contraction takes place only along the direction of motion.

QUICK QUIZ 26.2

You are packing for a trip to another star, to which you will be traveling at $0.99c$. Should you buy smaller sizes of your clothing, because you will be skinnier on the trip? Can you sleep in a smaller cabin than usual, because you will be shorter when you lie down?

QUICK QUIZ 26.2 ANSWER

The answers to both of these questions is **no**. Both your clothing and your sleeping cabin are at rest in your reference frame, thus, they will have their proper length. There will be no change in measured lengths of objects within your spacecraft. Another observer, on a spacecraft traveling at a high speed relative to yours, will measure you as thinner (if your body is oriented in a direction perpendicular to the direction of motion relative to him) or will claim that you are able to fit into a shorter sleeping cabin (if your body is oriented in a direction parallel to your direction of travel relative to the other observer).
QUICK QUIZ 26.3

You are observing a rocket moving away from you. Compared to its length when it was at rest on the ground, you will measure its length to be (a) shorter, (b) longer, or (c) the same. Now you see a clock through a window on the rocket. Compared to the passage of time measured by the watch on your wrist, you observe that the passage of time on the rocket's clock is (d) faster, (e) slower, or (f) the same. Answer the same questions if the rocket turns around and comes toward you.

QUICK QUIZ 26.3 ANSWER

(a), (e). The outgoing rocket will appear to have a shorter length and a slower clock. The answers are the same for the incoming rocket. Length contraction and time dilation depend only on the magnitude of the relative velocity, not on the direction.

Relativistic Definitions

- To properly describe the motion of particles within special relativity, Newton’s laws of motion and the definitions of momentum and energy need to be generalized.
- These generalized definitions reduce to the classical ones when the speed is much less than c.

Relativistic Momentum

- To account for conservation of momentum in all inertial frames, the definition must be modified.
  \[ p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv \]
  - \( \gamma \) is the Lorentz factor.
  - \( m \) is the mass of the particle when it is at rest.
  - \( v \) is the speed of the particle.
  - \( c \) is the speed of light.
  - When the speed of the particle is much less than the speed of light, \( \gamma \approx 1 \) and the relativistic momentum reduces to the classical momentum.
  - When the speed of the particle is close to the speed of light, \( \gamma \gg 1 \) and the relativistic momentum is significantly larger than the classical momentum.

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Relativistic Addition of Velocities

- Galilean relative velocities cannot be applied to objects moving near the speed of light.
- Einstein’s modification is:

\[ v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad}v_{db}}{c^2}} \]

- The denominator is a correction based on length contraction and time dilation.

Relativistic Corrections

- Remember, relativistic corrections are needed because no material objects can travel faster than the speed of light.

Relativistic Energy

- The definition of kinetic energy requires modification in relativistic mechanics.

\[ KE = \gamma mc^2 - mc^2 \]

- The term \( mc^2 \) is called the rest energy of the object and is independent of its speed.
- The term \( \gamma mc^2 \) is the total energy, \( E \), of the object and depends on its speed and its rest energy.

Relativistic Energy – Consequences

- A particle has energy by virtue of its mass alone.
- A stationary particle with zero kinetic energy has an energy proportional to its inertial mass.
- The mass of a particle may be completely convertible to energy and pure energy may be converted to particles.
Energy and Relativistic Momentum

- It is useful to have an expression relating total energy, $E$, to the relativistic momentum, $p$
  - $E^2 = p^2c^2 + (mc^2)^2$
    - When the particle is at rest, $p = 0$ and $E = mc^2$
    - Massless particles ($m = 0$) have $E = pc$
  - This is also used to express masses in energy units
    - Mass of an electron = $9.11 \times 10^{-31}$ kg = 0.511 Me
    - Conversion: 1 u = 929.494 MeV/c^2

Quick Quiz 26.4

A photon is reflected from a mirror. **True or false:**
(a) Because a photon has a zero mass, it does not exert a force on the mirror.  
(b) Although the photon has energy, it cannot transfer any energy to the surface because it has zero mass.  
(c) The photon carries momentum, and when it reflects off the mirror, it undergoes a change in momentum and exerts a force on the mirror.  
(d) Although the photon carries momentum, its change in momentum is zero when it reflects from the mirror, so it cannot exert a force on the mirror.

Quick Quiz 26.4 Answer

(a) False  
(b) False  
(c) True  
(d) False

A reflected photon does exert a force on the surface. Although a photon has zero mass, a photon does carry momentum. When it reflects from a surface, there is a change in the momentum, just like the change in momentum of a ball bouncing off a wall. According to the momentum interpretation of Newton’s second law, a change in momentum results in a force on the surface. This concept is used in theoretical studies of space sailing. These studies propose building nonpowered spacecraft with huge reflective sails oriented perpendicularly to the rays from the Sun. The large number of photons from the Sun reflecting from the surface of the sail will exert a force which, although small, will provide a continuous acceleration. This would allow the spacecraft to travel to other planets without fuel.

Pair Production

- An electron and a positron are produced and the photon disappears
  - A positron is the antiparticle of the electron, same mass but opposite charge
- Energy, momentum, and charge must be conserved during the process
- The minimum energy required is $2m_e = 1.04$ MeV
Pair Annihilation

- In pair annihilation, an electron-positron pair produces two photons
  - The inverse of pair production
- It is impossible to create a single photon
  - Momentum must be conserved