Lecture 15

- Modern Physics
  1. Quantum Physics
     - The Compton Effect
     - Photons and EM Waves
     - Wave Properties of Particles
     - Wave Functions
     - The Uncertainty Principle

http://www.physics.wayne.edu/~alan/2140Website/Main.htm
Reminder: Exam 3
Friday, July 6

- 12-13 questions.
- Show your work for credit.
- Closed book.
- You may bring a page of notes.
- Bring a calculator and a pen or pencil
Review Problem: A xenon arc lamp is covered with an interference filter that only transmits light of 400-nm wavelength. When the transmitted light strikes a metal surface, a stream of electrons emerges from the metal. If the intensity of the light striking the surface is doubled,

1. more electrons are emitted in a given time interval.
2. the electrons that are emitted are more energetic.
3. both of the above.
4. neither of the above.
The Compton Effect

- Compton directed a beam of x-rays toward a block of graphite.
- He found that the scattered x-rays had a slightly longer wavelength than the incident x-rays.
  - This means they also had less energy.
- The amount of energy reduction depended on the angle at which the x-rays were scattered.
- The change in wavelength is called the *Compton shift*. 
Compton Scattering

- Compton assumed the photons acted like other particles in collisions
- Energy and momentum were conserved
- The shift in wavelength is

\[ \Delta \lambda = \lambda - \lambda_o = \frac{h}{m_e c} (1 - \cos \theta) \]

Compton wavelength = 0.00243 nm
Compton Scattering

- The quantity $\frac{h}{m_e c}$ is called the *Compton wavelength*
  - Compton wavelength = 0.00243 nm
  - Very small compared to visible light
- The Compton shift depends on the *scattering angle* and not on the *wavelength*
- Experiments confirm the results of Compton scattering and strongly support the photon concept
A beam of 0.68-nm photons (E=1828 eV) undergoes Compton scattering from free electrons. What are the energy and momentum of the photons that emerge at a 45° angle with respect to the incident beam?

\[ \Delta \lambda = \lambda - \lambda_o = \frac{h}{m_e c} (1 - \cos \theta) \]

\[ \Delta \lambda = 0.00243 \text{ nm} \times (1 - 0.707) = 7.11 \times 10^{-4} \text{ nm} \]

\[ E = \frac{hc}{\lambda} = \frac{hc}{0.6807 \text{ nm}} = 1826 \text{ eV} \]

\[ p = \frac{h}{\lambda} = \frac{h}{0.6807 \text{ nm}} = 1826 \text{ eV/c} \]
QUICK QUIZ 1

An x-ray photon is scattered by an electron. The frequency of the scattered photon relative to that of the incident photon (a) increases, (b) decreases or (c) remains the same.

(b). Some energy is transferred to the electron in the scattering process. Therefore, the scattered photon must have less energy (and hence, lower frequency) than the incident photon.
A photon of energy $E_0$ strikes a free electron, with the scattered photon of energy $E$ moving in the direction opposite that of the incident photon. In this Compton effect interaction, the resulting kinetic energy of the electron is (a) $E_0$, (b) $E$, (c) $E_0 - E$, (d) $E_0 + E$, (e) none of the above.

(c). Conservation of energy requires the kinetic energy given to the electron be equal to the difference between the energy of the incident photon and that of the scattered photon.
In 1924, Louis de Broglie postulated that because photons have wave and particle characteristics, perhaps all forms of matter have both properties.

For instance, for a photon:

$$E = hf = \frac{hc}{\lambda}$$

thus

$$p = \frac{E}{c} = \frac{hc}{c\lambda} = \frac{h}{\lambda}$$

or

$$\lambda = \frac{h}{p}$$

De Broglie suggested that this formula is true for any particle! Thus, the frequency and wavelength of matter waves can be determined. I.e. de Broglie wavelength of a particle is

$$\lambda = \frac{h}{mv}$$
Wave Properties of Particles

- The frequency of matter waves can also be determined.
- De Broglie postulated that all particles satisfy Einstein’s relation
  \[ E = hf \]
- Or, in other words,
  \[ f = \frac{E}{h} \]
The Davisson-Germer Experiment

- They scattered low-energy electrons from a nickel target.
- They followed this with extensive *diffraction measurements* from various materials.
- The wavelength of the electrons calculated from the diffraction data agreed with the expected de Broglie wavelength.
- This confirmed the wave nature of electrons.
- Other experimenters have confirmed the wave nature of other particles.
Problem: the wavelength of a proton

Calculate the de Broglie wavelength for a proton \((m_p=1.67\times10^{-27} \text{ kg})\) moving with a speed of \(1.00 \times 10^7 \text{ m/s}\).
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**Given:**

\( v = 1.0 \times 10^7 \text{ m/s} \)

**Find:**

\( \lambda_p = ? \)

Given the velocity and a mass of the proton we can compute its wavelength

\[
\lambda_p = \frac{h}{m_p v}
\]

Or numerically,

\[
\lambda_{ps} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(1.67 \times 10^{-31} \text{ kg})(1.00 \times 10^7 \text{ m/s})} = 3.97 \times 10^{-14} \text{ m}
\]
A non-relativistic electron and a non-relativistic proton are moving and have the same de Broglie wavelength. Which of the following are also the same for the two particles: (a) speed, (b) kinetic energy, (c) momentum, (d) frequency?

(c). Two particles with the same de Broglie wavelength will have the same momentum \( p = mv \). If the electron and proton have the same momentum, they cannot have the same speed because of the difference in their masses. For the same reason, remembering that \( KE = \frac{p^2}{2m} \), they cannot have the same kinetic energy. Because the kinetic energy is the only type of energy an isolated particle can have, and we have argued that the particles have different energies, Equation 27.15 \( f = \frac{E}{h} \) tells us that the particles do not have the same frequency.
The Electron Microscope

- The electron microscope depends on the wave characteristics of electrons.
- Microscopes can only resolve details that are slightly smaller than the wavelength of the radiation used to illuminate the object.
- The electrons can be accelerated to high energies and have small wavelengths.

\[ \lambda_{e^-} \approx 5 \times 10^{-12} \text{ m (5 pm)} \] for 50 kV acceleration potential.
27.8 The Wave Function

In 1926 Schrödinger proposed a wave equation that describes the manner in which matter waves change in space and time.

Schrödinger’s wave equation is a key element in quantum mechanics.

\[ i \frac{\Delta \Psi}{\Delta t} = H \Psi \]

Schrödinger’s wave equation is generally solved for the wave function, \( \Psi \).
The Wave Function

- The wave function depends on the particle’s position and the time.

- The value of $|\Psi|^2$ at some location at a given time is proportional to the probability of finding the particle at that location at that time.
Orbitals of Atomic Hydrogen

Computer generated figures of atomic orbitals (electron wave functions) for the Hydrogen atom.
27.9 The Uncertainty Principle

When measurements are made, the experimenter is always faced with experimental uncertainties in the measurements

- Classical mechanics offers no fundamental barrier to ultimate refinements in measurements
- Classical mechanics would allow for measurements with arbitrarily small uncertainties
The Uncertainty Principle

Quantum mechanics predicts that a barrier to measurements with ultimately small uncertainties does exist.

In 1927 Heisenberg introduced the *uncertainty principle*

- If a measurement of position of a particle is made with precision $\Delta x$ and a simultaneous measurement of linear momentum is made with precision $\Delta p$, then the product of the two uncertainties can never be smaller than $h/4\pi$. 

The Uncertainty Principle

Mathematically,

\[ \Delta x \Delta p_x \geq \frac{h}{4\pi} \]

It is physically impossible to measure simultaneously the exact position and the exact linear momentum of a particle.

Another form of the principle deals with energy and time:

\[ \Delta E \Delta t \geq \frac{h}{4\pi} \]
A thought experiment for viewing an electron with a powerful microscope.

In order to see the electron, at least one photon must bounce off it.

During this interaction, momentum is transferred from the photon to the electron.

Therefore, the light that allows you to accurately locate the electron changes the momentum of the electron.
Problem: macroscopic uncertainty

A 50.0-g ball moves at 30.0 m/s. If its speed is measured to an accuracy of 0.10%, what is the minimum uncertainty in its position?
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**Given:**

\[ \begin{align*} 
v &= 30 \text{ m/s} \\
\Delta v/v &= 0.10\% \\
m &= 50.0 \text{ g} 
\end{align*} \]

**Find:**

\[ \delta x = ? \]

Notice that the ball is non-relativistic. Thus, \( p = mv \), and uncertainty in measuring momentum is

\[
\Delta p = m \left( \frac{\delta v}{v} \right) = m \left( \frac{\Delta v}{v} \cdot v \right) = \left( 50.0 \times 10^{-2} \text{ kg} \right) \left( 1.0 \times 10^{-3} \cdot 30 \text{ m/s} \right) = 1.5 \times 10^{-2} \text{ kg} \cdot \text{m/s}
\]

Thus, uncertainty relation implies

\[
\Delta x \geq \frac{\hbar}{4\pi \left( \Delta p \right)} = \frac{6.63 \times 10^{-24} \text{ J} \cdot \text{s}}{4\pi \left( 1.5 \times 10^{-3} \text{ kg} \cdot \text{m/s} \right)} = 3.5 \times 10^{-32} \text{ m}
\]
Problem: Macroscopic measurement

A 0.50-kg block rests on the icy surface of a frozen pond, which we can assume to be frictionless. If the location of the block is measured to a precision of 0.50 cm, what speed must the block acquire because of the measurement process?

Recall: \[ \Delta x \Delta p_x \geq \frac{h}{4\pi} \] and \[ p = mv \]
Scanning Tunneling Microscope (STM)

- Allows highly detailed images with resolution comparable to the size of a single atom
- A conducting probe with a sharp tip is brought near the surface
- The electrons can “tunnel” across the barrier of empty space
Scanning Tunneling Microscope, cont

- By applying a voltage between the surface and the tip, the electrons can be made to tunnel preferentially from surface to tip.
- The tip samples the distribution of electrons just above the surface.
- The STM is very sensitive to the distance between the surface and the tip.
  - Allows measurements of the height of surface features within 0.001 nm.
Limitation of the STM

There is a serious limitation to the STM since it depends on the conductivity of the surface and the tip:

- Most materials are not conductive at their surface.
- An *atomic force microscope* has been developed that overcomes this limitation.
- It measures the force between the tip and the sample surface.
- Has comparable sensitivity.
More STM Images