Lecture 6

- Electrodynamics
  - Direct current circuits
  - parallel and series connections
  - Kirchhoff’s rules
  - RC circuits

http://www.physics.wayne.edu/~alan/2140Website/Main.htm

Chapter 18

Lightning Review

Last lecture:

1. Current and resistance
   \[ I = \frac{\Delta Q}{\Delta t} \]
   - Temperature dependence of resistance
     \[ R = R_0 \left[ 1 + \alpha (T - T_0) \right] \]
   - Power in electric circuits
     \[ P = I \Delta V = I^2 R = \frac{(\Delta V)^2}{R} \]

2. DC Circuits
   - EMF
     \[ \Delta V = E - Ir \]
   - Resistors in series
     \[ R_{eq} = R_1 + R_2 + R_3 + ... \]

Introduction: elements of electrical circuits

- **A branch**: A branch is a single electrical element or device (resistor, etc.).

- **A junction**: A junction (or node) is a connection point between two or more branches.

- If we start at any point in a circuit (node), proceed through connected electric devices back to the point (node) from which we started, without crossing a node more than one time, we form a closed-path (or loop).
18.1 Sources of EMF

- **Steady current** (constant in magnitude and direction)
  - requires a complete circuit
  - path cannot be only resistance
  - cannot be only potential drops in direction of current flow
- Electromotive Force (EMF)
  - provides *increase* in potential $\mathcal{E}$
  - converts some external form of energy into electrical energy
- Single emf and a single resistor: emf can be thought of as a “charge pump” $V = IR$

\[ V = IR = \mathcal{E} \]

**EMF (continued)**

- Now add a load resistance $R$
- Since it is connected by a conducting wire to the battery → terminal voltage is the same as the potential difference across the load resistance
  \[ \Delta V = \mathcal{E} - Ir = IR, \text{ or } \mathcal{E} = Ir + IR \]
- Thus, the current in the circuit is
  \[ I = \frac{\mathcal{E}}{R+r} \]

\[ I\mathcal{E} = Ir + IrR \]

**Note:** we’ll assume $r$ negligible unless otherwise is stated

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**EMF**

- Each real battery has some internal resistance
- $AB$: potential increases by $\mathcal{E}$ on the source of EMF, then decreases by $Ir$ (because of the internal resistance)
- Thus, terminal voltage on the battery $\Delta V$ is
  \[ \Delta V = \mathcal{E} - Ir \]

**Note:** $\mathcal{E}$ is the same as the terminal voltage when the current is zero (open circuit)

**Measurements in electrical circuits**

**Voltmeters** measure Potential Difference (or voltage) across a device by being placed in parallel with the device.

**Ammeters** measure current through a device by being placed in series with the device.
Direct Current Circuits

- Two Basic Principles:
  - Conservation of Charge
  - Conservation of Energy

- Resistance Networks
  - Ohm’s Law: \( V_{ab} = IR_{eq} \)
  - \( R_{eq} = \frac{V_{ab}}{I} \)

Resistors in series: notes

- Analogous formula is true for any number of resistors,
  \( R_{eq} = R_1 + R_2 + R_3 + \ldots \)  (series combination)

- It follows that the equivalent resistance of a series combination of resistors is greater than any of the individual resistors

Resistors in series: example

In the electrical circuit below, find voltage across the resistor \( R_1 \) in terms of the resistances \( R_1, R_2 \) and potential difference between the battery’s terminals \( V \).

Energy conservation implies:

- \( V = V_1 + V_2 \) with \( V_1 = IR_1 \) and \( V_2 = IR_2 \)

Then,

- \( V = I(R_1 + R_2) \), so \( I = \frac{V}{R_1 + R_2} \)

Thus,

- \( V_1 = V \cdot \frac{R_1}{R_1 + R_2} \)

This circuit is known as voltage divider.
18.3 Resistors in parallel

1. Since both R₁ and R₂ are connected to the same battery, potential differences across R₁ and R₂ are the same.

\[ V₁ = V₂ = V \]

2. Because of the charge conservation, current entering the junction A must equal the current leaving this junction, 

\[ I = I₁ + I₂ \]

By definition,

\[ I = \frac{V}{R_{eq}} \]

Thus, R_{eq} would be 

\[ \frac{1}{R_{eq}} = \frac{1}{R₁} + \frac{1}{R₂} \]

or 

\[ R_{eq} = \frac{R₁R₂}{R₁ + R₂} \]

Resistors in parallel: notes

- Analogous formula is true for any number of resistors,

\[ \frac{1}{R_{eq}} = \frac{1}{R₁} + \frac{1}{R₂} + \frac{1}{R₃} + \ldots \] (parallel combination)

- It follows that the equivalent resistance of a parallel combination of resistors is always less than any of the individual resistors

Resistors in parallel: example

In the electrical circuit below, find current through the resistor R₁ in terms of the resistances R₁, R₂ and total current I induced by the battery.

Charge conservation implies:

\[ I = I₁ + I₂ \]

with 

\[ I₁ = \frac{V}{R₁}, \text{ and } I₂ = \frac{V}{R₂} \]

Then,

\[ I₁ = \frac{IR₁}{R₂}, \text{ with } R_{eq} = \frac{R₁R₂}{R₁ + R₂} \]

Thus,

\[ I₁ = I \cdot \frac{R₂}{R₁ + R₂} \]

This circuit is known as current divider.

Direct current circuits: example

Find the currents I₁ and I₂ and the voltage Vₓ in the circuit shown below.

Strategy:

1. Find current I by finding the equivalent resistance of the circuit
2. Use current divider rule to find the currents I₁ and I₂
3. Knowing I₂, find Vₓ.
Direct current circuits: example

Find the currents \( I_1 \) and \( I_2 \) and the voltage \( V_x \) in the circuit shown below.

First find the equivalent resistance seen by the 20 V source:

\[
R_{eq} = \frac{7 \Omega + \frac{4 \Omega(12 \Omega)}{12 \Omega + 4 \Omega}}{10 \Omega}
\]

Then find current \( I \) by,

\[
I = \frac{20V}{R_{eq}} = \frac{20V}{10\Omega} = 2 A
\]

Then find current \( I_1 \) by,

\[
I_1 = \frac{2A(4\Omega)}{12\Omega + 4\Omega} = 0.5 A, \text{ and } I_2 = I - I_1 = 1.5 A
\]

Finally, voltage \( V_x \) is

\[
V_x = I_2(4\Omega) = 1.5A(4\Omega) = 6V
\]

Example:

Determine the equivalent resistance of the circuit as shown.
Determine the voltage across and current through each resistor.
Determine the power dissipated in each resistor
Determine the power delivered by the battery

\[
E=18V
\]

\[
R_1=4\Omega, R_2=3\Omega, R_3=6\Omega
\]

\[
R_{eq}=6\Omega
\]

So, \( I_{eq} = \frac{E}{R_{eq}} = 3A \)

\[
P=I_{eq}E = 108W
\]

18.4 Kirchhoff’s rules and DC currents

- The procedure for analyzing complex circuits is based on the principles of conservation of charge and energy
- They are formulated in terms of two Kirchhoff’s rules:
  1. The sum of currents entering any junction must equal the sum of the currents leaving that junction (current or junction rule).
  2. The sum of the potential differences across all the elements around any closed-circuit loop must be zero (voltage or loop rule).
1. Junction rule

As a consequence of the Law of the conservation of charge, we have:

- The sum of the currents entering a node (junction point) equal to the sum of the currents leaving.

\[ I_a + I_b = I_c + I_d \]

Similar to the water flow in a pipe.

1a, 1b, 1c, and 1d can each be either a positive or negative number.

2. Loop rule

As a consequence of the Law of the conservation of energy, we have:

- The sum of the potential differences across all the elements around any closed loop must be zero.

1. Assign symbols and directions of currents in the loop
   - If the direction is chosen wrong, the current will come out with the correct magnitude, but a negative sign (it's ok).

2. Choose a direction (cw or ccw) for going around the loop.
   - Record drops and rises of voltage according to this:
     - If a resistor is traversed in the direction of the current: \[ V = -IR \]
     - If a resistor is traversed in the direction opposite to the current: \[ V = +IR \]
     - If \( \varepsilon \) is traversed "from + to -": \[ V = +\varepsilon \]
     - If \( \varepsilon \) is traversed "from - to +": \[ V = -\varepsilon \]

Simplest Loop Rule Example:

- Single loop, start at point A
- Battery traversed from - to +
  - So use + \( \varepsilon \) (a voltage gain)
- Resistor traversed from + to -
  - So use -IR (a voltage drop)

For the loop we have:

\[ 0 = +\varepsilon - IR \]
Loop rule: illustration

Using sum of the drops = 0

Blue path, starting at “a”
+ V_1 - V_10 + V_9 - V_8 = 0

Red path, starting at “b”
+ V_2 + V_3 + V_8 - V_9 + V_{11} + V_{12} - V_1 = 0

Yellow path, starting at “b”
- V_2 + V_3 + V_8 - V_{10} + V_{11} + V_{12} - V_1 = 0

Kirchhoff’s Rules: Single-loop circuits

Example: For the circuit below find I, V_1, V_2, V_3, V_4 and the power supplied by the 10 volt source.

1. For convenience, we start at point “a” and sum voltage drops = 0 in the direction of the current I.

-10 - V_1 + 30 - V_3 - V_4 + 20 - V_2 = 0  \quad (1)

2. We note that: V_1 = 20I, V_2 = 40I, V_3 = 15I, V_4 = 5I  \quad (2)

3. We substitute the above into Eq. 1 to obtain Eq. 3 below.

-10 - 20I + 30 - 15I - 5I + 20 - 40I = 0  \quad (3)

Solving this equation gives, I = 0.5 A

Using this value of I in Eq. 2 gives:

V_1 = 10 V
V_2 = 20 V
V_3 = 7.5 V
V_4 = 2.5 V

P_{10\text{vollage}} = -10I = -5 W

Kirchhoff’s Rules: Single-loop circuits (cont.)

Charge across capacitor Q

Since capacitor is charging,
\[ q = Q \left( 1 - e^{-t/(RC)} \right) \]

As capacitor becomes charged, current slows because voltage across resistor is \( E - V_C \) and \( V_C \) gradually approaches \( E \).

Once capacitor is charged, current is zero.

RC is called the time constant.
Discharging the capacitor in RC circuit

- If a capacitor is charged and the switch is closed, then current flows and the voltage on the capacitor gradually decreases.

This leads to decreasing charge: \( q = Qe^{-t/RC} \)

Example: charging an unknown capacitor

A series combination of a 12 kΩ resistor and an unknown capacitor is connected to a 12 V battery. One second after the circuit is completed, the voltage across the capacitor is 10 V. Determine the capacitance of the capacitor.

**Given:**
- \( R = 12 \text{ kΩ} \)
- \( \varepsilon = 12 \text{ V} \)
- \( V = 10 \text{ V} \)
- \( t = 1 \text{ sec} \)

**Find:**
- \( C = ? \)

Recall that the charge is building up according to:

\[ q = Q(1 - e^{-t/RC}) \]

Thus the voltage across the capacitor changes as:

\[ V = \frac{q}{C} = \frac{Q}{C}(1 - e^{-t/RC}) = \varepsilon(t - e^{-t/RC}) \]

This is also true for voltage at \( t = 1s \) after the switch is closed,

\[ \frac{V}{\varepsilon} = 1 - e^{-t/RC} \Rightarrow e^{-t/RC} = 1 - \frac{V}{\varepsilon} \Rightarrow \frac{-t}{RC} = \log \left( 1 - \frac{V}{\varepsilon} \right) \]

\[ C = \frac{t}{R \log \left( 1 - \frac{V}{\varepsilon} \right)} = \frac{1}{(12,000 \text{ Ω}) \log \left( \frac{10 \text{ V}}{12 \text{ V}} \right)} = 46.5 \mu F \]