PHY 2130 Homework solutions
Assignment 15

23.7 The radius of curvature of a concave mirror is positive, so $R = +20.0\text{ cm}$. The mirror equation then gives

$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{p} = \frac{p - 10.0\text{ cm}}{(10.0\text{ cm})p}, \text{ or } q = \frac{(10.0\text{ cm})p}{p - 10.0\text{ cm}}.$$

(a) If $p = 40.0\text{ cm}$, $q = +13.3\text{ cm}$ and $M = -\frac{q}{p} = -\frac{13.3\text{ cm}}{40.0\text{ cm}} = \boxed{-0.333}$

The image is $\boxed{\text{13.3 cm in front of the mirror, real, and inverted}}$

(b) When $p = 20.0\text{ cm}$, $q = +20.0\text{ cm}$ and $M = -\frac{q}{p} = -\frac{20.0\text{ cm}}{20.0\text{ cm}} = \boxed{-1.00}$

The image is $\boxed{\text{20.0 cm in front of the mirror, real, and inverted}}$

(c) If $p = 10.0\text{ cm}$, $q = \frac{(10.0\text{ cm})(10.0\text{ cm})}{10.0\text{ cm} - 10.0\text{ cm}} \to \infty$,

and $\boxed{\text{no image is formed. Parallel rays leave the mirror}}$

23.18 The magnified, *real* images formed by concave mirrors are inverted. Thus, $M < 0$ giving

$$M = -\frac{q}{p} = -4, \text{ or } q = 4p$$

Then, $\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{p} + \frac{1}{4p} = \frac{5}{4p}$ or $R = \frac{8}{5}p = \frac{8}{5}(30.0\text{ cm}) = \boxed{48.0\text{ cm}}$

23.29 From the thin lens equation, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, the image distance is found to be

$$q = \frac{fp}{p - f} = \frac{(20.0\text{ cm})p}{p - 20.0\text{ cm}}$$

(a) If $p = 40.0\text{ cm}$, then $q = 40.0\text{ cm}$ and $M = -\frac{q}{p} = -\frac{40.0\text{ cm}}{40.0\text{ cm}} = \boxed{-1.00}$

The image is $\boxed{\text{real, inverted, and 40.0 cm beyond the lens}}$

(b) If $p = 20.0\text{ cm}$, $q \to \infty$ $\boxed{\text{No image formed. Parallel rays leave the lens.}}$

(c) When $p = 10.0\text{ cm}$, $q = -20.0\text{ cm}$ and

$$M = -\frac{q}{p} = -\frac{(-20.0\text{ cm})}{10.0\text{ cm}} = \boxed{+2.00}$$

The image is $\boxed{\text{virtual, upright, and 20.0 cm in front of the lens}}$

23.38 (a) The total distance from the object to the real image is the object-to-screen distance, so $p + q = 5.00\text{ m}$ or $q = 5.00\text{ m} - p$. The thin lens equation then becomes

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{5.00\text{ m} - p} = \frac{5.00\text{ m}}{p(5.00\text{ m} - p)},$$

or $p^2 - (5.00\text{ m})p + (5.00\text{ m})f = 0$

With $f = 0.800\text{ m}$, this gives $p^2 - (5.00\text{ m})p + 4.00\text{ m}^2 = 0$ which factors as $(p - 4.00\text{ m})(p - 1.00\text{ m}) = 0$ with two solutions:

$$\boxed{p = 4.00\text{ m and } p = 1.00\text{ m}}$$

(b) If $p = 4.00\text{ m}$, then $q = 1.00\text{ m}$ and $M = -\frac{q}{p} = -\frac{1.00\text{ m}}{4.00\text{ m}} = -\frac{1}{4}$

In this case, the image is $\boxed{\text{real, inverted, and one-quarter the size of the object}}$

If $p = 1.00\text{ m}$, then $q = 4.00\text{ m}$ and $M = -\frac{q}{p} = -\frac{4.00\text{ m}}{1.00\text{ m}} = -4$. In this case, the image is $\boxed{\text{real, inverted, and four times the size of the object}}$

23.41 The thin lens equation gives the image position for the first lens as

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(30.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm} - 15.0\text{ cm}} = +30.0\text{ cm},$$

and the magnification by this lens is $M_1 = -\frac{q_1}{p_1} = -\frac{30.0\text{ cm}}{30.0\text{ cm}} = -1.00$

The real image formed by the first lens serves as the object for the second lens, so $p_2 = 40.0\text{ cm} - q_1 = +10.0\text{ cm}$. Then, the thin lens equation gives

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(10.0\text{ cm})(15.0\text{ cm})}{10.0\text{ cm} - 15.0\text{ cm}} = -30.0\text{ cm}$$

and the magnification by the second lens is

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-30.0\text{ cm})}{10.0\text{ cm}} = +3.00$$

Thus, the final, virtual image is located $\boxed{30.0\text{ cm in front of the second lens}}$

and the overall magnification is $M = M_1 M_2 = (-1.00)(+3.00) = \boxed{-3.00}$

24.3 (a) The distance between the central maximum and the first order bright fringe is $\Delta y = y_{bright}\big|_{m=1} - y_{bright}\big|_{m=0} = \frac{\lambda L}{d}$, or

$$\Delta y = \frac{\lambda L}{d} = \frac{(546.1\times 10^{-9}\text{ m})(1.20\text{ m})}{0.250\times 10^{-3}\text{ m}} = 2.62\times 10^{-3}\text{ m} = \boxed{2.62\text{ mm}}$$

(b) The distance between the first and second dark bands is

$$\Delta y = y_{dark}\big|_{m=1} - y_{dark}\big|_{m=0} = \frac{\lambda L}{d} = \boxed{2.62\text{ mm}}\text{ as in (a) above.}$$

24.11 The distance between the central maximum (position of A) and the first minimum is

$$y = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right)\Big|_{m=0} = \frac{\lambda L}{2d}.$$

Thus, $d = \frac{\lambda L}{2y} = \frac{(3.00\text{ m})(150\text{ m})}{2(20.0\text{ m})} = \boxed{11.3\text{ m}}$

24.30 (a) Dark bands occur where $\sin\theta = m(\lambda/a)$. At the first dark band, $m = 1$ and the distance from the center of the central maximum is

$$y_1 = L\tan\theta \approx L\sin\theta = L\left(\frac{\lambda}{a}\right)$$

$$= (1.5\text{ m})\left(\frac{600\times 10^{-9}\text{ m}}{0.40\times 10^{-3}\text{ m}}\right) = 2.25\times 10^{-3}\text{ m} = \boxed{2.3\text{ mm}}$$

(b) The width of the central maximum is $2y_1 = 2(2.25\text{ mm}) = \boxed{4.5\text{ mm}}$