PHY 2130 Homework solutions
Assignment 7

7.5 (a) $\alpha = \frac{\left(2.51 \times 10^4 \text{ rev/min} - 0\right)}{3.20 \text{ s}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right) = \boxed{821 \text{ rad/s}^2}$

(b) $\theta = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\left(821 \frac{\text{rad}}{\text{s}^2}\right)(3.20 \text{ s})^2 = \boxed{4.21 \times 10^3 \text{ rad}}$

7.17 The final angular velocity is $\omega_f = 78 \frac{\text{rev}}{\text{min}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 8.17 \text{ rad/s}$,

and the radius of the disk is $r = 5.0 \text{ in}\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right) = 12.7 \text{ cm} = 0.127 \text{ m}$.

(a) The tangential acceleration of the bug as the disk speeds up is

$$a_t = r\alpha = r\left(\frac{\Delta\omega}{\Delta t}\right) = (0.127 \text{ m})\left(\frac{8.17 \text{ rad/s}}{3.0 \text{ s}}\right) = \boxed{0.35 \text{ m/s}^2}.$$

(b) The final tangential speed of the bug is

$$v_t = r\omega_f = (0.127 \text{ m})(8.17 \text{ rad/s}) = \boxed{1.0 \text{ m/s}}.$$

(c) At $t = 1.0$ s, $\omega = \omega_i + \alpha t = 0 + \left(\frac{8.17 \text{ rad/s}}{3.0 \text{ s}}\right)(1.0 \text{ s}) = 2.7 \text{ rad/s}$.

Thus, $a_t = r\alpha = \boxed{0.35 \text{ m/s}^2}$ as above, while the radial acceleration is

$$a_c = r\omega^2 = (0.127 \text{ m})(2.7 \text{ rad/s})^2 = \boxed{0.94 \text{ m/s}^2}.$$

The total acceleration is $a = \sqrt{a_c^2 + a_t^2} = \boxed{1.0 \text{ m/s}^2}$, and the angle this acceleration makes with the direction of $\mathbf{a}_c$ is

$$\theta = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{0.35}{0.94}\right) = \boxed{20^\circ}.$$

7.21 Friction must supply the needed centripetal force. Hence, it is necessary that

$$F_c \le (f_s)_{max}\text{, or } m\frac{v_t^2}{r} \le \mu_s(mg) \text{ and the maximum safe speed is}$$

$$(v_t)_{max} = \sqrt{\mu_s r g} = \sqrt{(0.70)(20\text{ m})(9.80\text{ m/s}^2)} = \boxed{12\text{ m/s}}.$$

7.25 (a) Since the 1.0-kg mass is in equilibrium, the tension in the string is

$$T = mg = (1.0\text{ kg})(9.8\text{ m/s}^2) = \boxed{9.8\text{ N}}.$$

(b) The tension in the string must produce the centripetal acceleration of the puck. Hence, $F_c = T = \boxed{9.8\text{ N}}$.

(c) From $F_c = m_{puck}\left(\frac{v_t^2}{r}\right)$, we find $v_t = \sqrt{\frac{rF_c}{m_{puck}}} = \sqrt{\frac{(1.0\text{ m})(9.8\text{ N})}{0.25\text{ kg}}} = \boxed{6.3\text{ m/s}}$.

7.35 (a) The gravitational force must supply the required centripetal acceleration, so

$$\frac{Gm_E m}{r^2} = m\left(\frac{v_t^2}{r}\right)\text{. This reduces to } r = \frac{Gm_E}{v_t^2}\text{, which gives}$$

$$r = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.98\times10^{24}\text{ kg})}{(5000\text{ m/s})^2} = 1.596\times10^7\text{ m}.$$

The altitude above the surface of the Earth is then

$$h = r - R_E = 1.596\times10^7\text{ m} - 6.38\times10^6\text{ m} = \boxed{9.58\times10^6\text{ m}}.$$

(b) The time required to complete one orbit is

$$T = \frac{\text{circumference of orbit}}{\text{orbital speed}} = \frac{2\pi(1.596\times10^7\text{ m})}{5000\text{ m/s}} = 2.00\times10^4\text{ s} = \boxed{5.57\text{ h}}.$$