Lecture 14

- Energy
  - Work
  - Kinetic energy

http://www.physics.wayne.edu/~apetrov/PHY2130/
Last lecture:

1. Rotational motion:
   - orbits, Kepler’s laws
   - motion with constant angular acceleration
   - apparent weight

Review Problem: If an astronaut has a mass of 90 kg on Earth, what is the astronaut’s mass on the Moon? The acceleration of gravity on the Moon is 1/6 of that on Earth.

(1) 15 kg
(2) 90 kg
(3) 147 kg
(4) 540 kg
Energy
Introduction

- Forms of energy:
  - Mechanical
    - focus for now
  - chemical
  - electromagnetic
  - nuclear

- Energy can be transformed from one form to another
  - Essential to the study of physics, chemistry, biology, geology, astronomy

- Can be used in place of Newton’s laws to simplify solution of certain problems
Notes About Conservation of Energy

- We can neither create nor destroy energy
  - Another way of saying energy is conserved
  - If the total energy of the system does not remain constant, the energy must have crossed the boundary by some mechanism
  - Applies to areas other than physics
Let’s try a movie!
Work

• Provides a link between force and energy

• The work, $W$, done by a constant force on an object is defined as the product of the component of the force along the direction of displacement and the magnitude of the displacement.

$$W \equiv (F \cos \theta) \Delta x$$

- $(F \cos \theta)$ is the component of the force in the direction of the displacement
- $\Delta x$ is the displacement
Work

- This gives no information about
  - the time it took for the displacement to occur
  - the velocity or acceleration of the object

Note: work is zero when
- there is no displacement (holding a bucket)
- force and displacement are perpendicular to each other, as \( \cos 90° = 0 \) (if we are carrying the bucket horizontally, gravity does no work)

\[
W \equiv (F \cos \theta) \Delta x
\]

(different from everyday “definition” of work)
More About Work

- **Scalar quantity**

<table>
<thead>
<tr>
<th>Units of Work</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>SI</td>
<td>joule (J=N m)</td>
</tr>
<tr>
<td>CGS</td>
<td>erg (erg=dyne cm)</td>
</tr>
<tr>
<td>US Customary</td>
<td>foot-pound (foot-pound=ft lb)</td>
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- If there are multiple forces acting on an object, the total work done is the algebraic sum of the amount of work done by each force.
More About Work

- Work can be **positive** or **negative**
  - Positive if the force and the displacement are **in the same direction**
  - Negative if the force and the displacement are **in the opposite direction**
- **Example 1**: lifting a cement block…
  - Work done by the person:
    - is positive when lifting the box
    - is negative when lowering the box
- **Example 2**: … then moving it horizontally
  - Work done by gravity:
    - is negative when lifting the box
    - is positive when lowering the box
    - is zero when moving it horizontally

**Total work**:

\[
W = W_1 + W_2 + W_3 = -mgh + mgh + 0 = 0
\]

(lifting  lowering  moving  total)
Example: A ball is tossed straight up. What is the work done by the force of gravity on the ball as it rises?

FBD for rising ball:

\[ W_g = w\Delta y \cos 180^\circ \]
\[ = -mg\Delta y \]

Note the sign: force and displacement point in the opposite directions!
Eric decided to clean his dorm room with his vacuum cleaner. While doing so, he pulls the canister of the vacuum cleaner with a force of magnitude \( F = 50.0 \, \text{N} \) at an angle \( 30.0^\circ \). He moves the vacuum cleaner a distance of 3.00 meters. Calculate the work done by all the forces acting on the canister.
Problem: cleaning the dorm room

Given:

- angle: $\alpha = 30^\circ$
- force: $F = 55.0 \text{ N}$
- displac.: $s = 3 \text{ m}$

Find:

- Work $W_F = ?$
- $W_n = ?$
- $W_{mg} = ?$

1. Introduce coordinate frame:
   - Oy: y is directed up
   - Ox: x is directed right

2. Note: horizontal displacement only,
   - Work: $W = (F \cos \theta) s$

\[ W_F = (F \cos \theta) s = (50.0 \text{ N})(\cos 30.0^\circ)(3.00\text{ m}) = 130 \text{ N} \cdot \text{m} = 130 \text{ J} \quad \checkmark \]

\[ W_n = (n \cos \theta) s = (n)(\cos 90.0^\circ)(3.00\text{ m}) = 0 \text{ J} \]

\[ W_{mg} = (mg \cos \theta) s = (n)(\cos(-90.0^\circ))(3.00\text{ m}) = 0 \text{ J} \]

No work as force is perpendicular to the displacement
Example: A box of mass $m$ is towed up a frictionless incline at constant speed. The applied force $F$ is parallel to the incline. What is the net work done on the box?

Apply Newton’s 2nd Law:

$$\sum F_x = F - w \sin \theta = 0$$
$$\sum F_y = N - w \cos \theta = 0$$
Example continued:

The magnitude of $F$ is:

$$F = mg \sin \theta$$

If the box travels along the ramp a distance of $\Delta x$ the work by the force $F$ is

$$W_F = F\Delta x \cos 0^\circ = mg\Delta x \sin \theta$$

The work by gravity is

$$W_g = w\Delta x \cos(\theta + 90^\circ) = -mg\Delta x \sin \theta$$
Example continued:

The work by the normal force is:

\[ W_N = N \Delta x \cos 90^\circ = 0 \]

The net work done on the box is:

\[ W_{\text{net}} = W_F + W_g + W_N \\
= mg \Delta x \sin \theta - mg \Delta x \sin \theta + 0 \\
= 0 \]
Graphical Representation of Work

- Split total displacement \((x_f - x_i)\) into many small displacements \(\Delta x\)
- For each small displacement:

\[
W_i = (F \cos \theta) \Delta x_i
\]

Thus, total work is:

\[
W_{tot} = \sum_i W_i = \sum_i F_x \cdot \Delta x_i
\]

which is total area under the \(F(x)\) curve!
Kinetic Energy

• Energy associated with the motion of an object
• Scalar quantity with the same units as work
• Work is related to kinetic energy
• Let $F$ be a constant force:

$$W_{net} = (F \cos \theta)s = (ma_x)s, \text{ but:}$$

$$v^2 = v_0^2 + 2a_x \cdot s, \text{ or } a_x \cdot s = \frac{v^2 - v_0^2}{2}.$$ 

Thus: $W_{net} = m\left(\frac{v^2 - v_0^2}{2}\right) = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$

This quantity is called kinetic energy:

$$KE = \frac{1}{2}mv^2$$
Work-Kinetic Energy Theorem

- When work is done by a net force on an object and the only change in the object is its speed, the work done is equal to the change in the object’s kinetic energy.

\[ W_{\text{net}} = KE_f - KE_i = \Delta KE \]

- Speed will increase if work is positive.
- Speed will decrease if work is negative.
Work and Kinetic Energy

- An object’s kinetic energy can also be thought of as the amount of work the moving object could do in coming to rest.
- The moving hammer has kinetic energy and can do work on the nail.
Two marbles, one twice as heavy as the other, are dropped to the ground from the roof of a building. Just before hitting the ground, the heavier marble has

1. as much kinetic energy as the lighter one.
2. twice as much kinetic energy as the lighter one.
3. half as much kinetic energy as the lighter one.
4. four times as much kinetic energy as the lighter one.
5. impossible to determine.
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Note: Kinetic energy is proportional to mass.
Example: The extinction of the dinosaurs and the majority of species on Earth in the Cretaceous Period (65 Myr ago) is thought to have been caused by an asteroid striking the Earth near the Yucatan Peninsula. The resulting ejecta caused widespread global climate change.

If the mass of the asteroid was $10^{16}$ kg (diameter in the range of 4-9 miles) and had a speed of 30.0 km/sec, what was the asteroid’s kinetic energy?

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \left(10^{16} \text{ kg}\right) \left(30 \times 10^3 \text{ m/s}\right)^2$$

$$= 4.5 \times 10^{24} \text{ J}$$

This is equivalent to $\sim 10^9$ Megatons of TNT.