Lecture 25

• Oscillations
  ➢ simple harmonic motion
  ➢ pendulum
  ➢ driven and damped oscillations

http://www.physics.wayne.edu/~apetrov/PHY2130/
Lightning Review

Last lecture:

1. Oscillations
   ✓ elastic energy, Hooke’s law
   ✓ simple harmonic motion

**Review Problem:** A figure skater is spinning at a rate of 1.4 rev/s with her arms outstretched. She then draws her arms in to her chest, reducing her rotational inertia to 61% of its original value. What is her new rate of rotation?

Note: HW Set 8, problem 3.
A figure skater is spinning at a rate of 1.4 rev/s with her arms outstretched. She then draws her arms in to her chest, reducing her rotational inertia to 61% of its original value. What is her new rate of rotation?
Simple Harmonic Motion and Uniform Circular Motion

- A ball is attached to the rim of a turntable of radius $A$
- The focus is on the shadow that the ball casts on the screen
- When the turntable rotates with a constant angular speed, the shadow moves in simple harmonic motion
Period and Frequency from Circular Motion

- **Period**
  \[ T = 2\pi \sqrt{\frac{m}{k}} \]

  - This gives the time required for an object of mass \( m \) attached to a spring of constant \( k \) to complete one cycle of its motion

- **Frequency**
  \[ f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

  - Units are cycles/second or Hertz, Hz

- The angular frequency is related to the frequency
  \[ \omega = 2\pi f = \sqrt{\frac{k}{m}} \]
Motion as a Function of Time

- Use of a *reference circle* allows a description of the motion
  - \( x = A \cos(2\pi ft) \)
    - \( x \) is the position at time \( t \)
    - \( x \) varies between \( +A \) and \( -A \)
Graphical Representation of SHM

- When $x$ is a maximum or minimum, velocity is zero.
- When $x$ is zero, the velocity is a maximum.
- When $x$ is a maximum in the positive direction, $a$ is a maximum in the negative direction.
\[ x(t) = A \cos \omega t \]

\[ v(t) = \frac{\Delta x}{\Delta t} = -A\omega \sin \omega t \]

\[ a(t) = \frac{\Delta v}{\Delta t} = -A\omega^2 \cos \omega t \]
Example: The period of oscillation of an object in an ideal mass-spring system is 0.50 sec and the amplitude is 5.0 cm. What is the speed at the equilibrium point?

Idea: let’s use energy conservation: at equilibrium $x = 0$:

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

Since $E=\text{constant}$, at equilibrium $(x = 0)$ the KE must be a maximum. Thus, $v = v_{\text{max}} = A\omega$.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.50 \text{ s}} = 12.6 \text{ rads/sec}$$

and $v = A\omega = (5.0 \text{ cm})(12.6 \text{ rads/sec}) = 62.8 \text{ cm/sec}$
Verification of Sinusoidal Nature

- This experiment shows the sinusoidal nature of simple harmonic motion
- The spring mass system oscillates in simple harmonic motion
- The attached pen traces out the sinusoidal motion
Example: A sewing machine needle moves with a rapid vibratory motion, rather like SHM, as it sews a seam. Suppose the needle moves 8.4 mm from its highest to its lowest position and it makes 24 stitches in 9.0 s. What is the maximum needle speed?

The maximum needle speed.

\[ v_m = \omega A = \frac{2\pi A}{T} = \frac{2\pi \left( \frac{8.4}{2} \times 10^{-3} \text{ m} \right)}{\frac{9.0 \text{ s}}{24}} = 7.0 \text{ cm/s} \]
The Simple Pendulum

- The simple pendulum is another example of simple harmonic motion
- The force is the component of the weight tangent to the path of motion

\[ F = -m g \sin \theta \]
Simple Pendulum

• In general, the motion of a pendulum is not simple harmonic

• However, for small angles, it becomes simple harmonic

  • In general, angles < 15° are small enough so \( \sin \theta = \theta \)

  \[ F = -mg \theta \]

• This force law looks like Hooke’s Law!
Simple Pendulum Compared to a Spring-Mass System
The Simple Pendulum

\[ \sum F_x = -T \sin \theta = -\frac{T_x}{L} = ma_x \]  \hspace{1cm} (1)

\[ \sum F_y = T \cos \theta - mg = ma_y \]  \hspace{1cm} (2)

When \( \theta \) is small,

\( \cos \theta = 1, \ a_y \approx 0 \)

Equation (2) \( \Rightarrow \ T = mg \)

Equation (1) \( \Rightarrow \ -\frac{mgx}{L} = ma_x \Rightarrow \ a_x = -\frac{g}{L}x \)

Motion is Simple Harmonic: \( a_x = -\omega^2 x \)

For small oscillations

\[ \omega = \sqrt{\frac{g}{L}} \quad T = 2\pi \sqrt{\frac{L}{g}} \]
Example: A clock has a pendulum that performs one full swing every 1.0 sec. The object at the end of the string weighs 10.0 N. What is the length of the pendulum?

Period of oscillation for pendulum depends on its length: 

\[ T = 2\pi \sqrt{\frac{L}{g}} \]

Solving for L:

\[ L = \frac{gT^2}{4\pi^2} = \frac{(9.8 \text{ m/s}^2)(1.0 \text{ s})^2}{4\pi^2} = 0.25 \text{ m} \]
Example: The gravitational potential energy of a pendulum is $U = mgy$. Taking $y = 0$ at the lowest point of the swing, show that $y = L(1 - \cos \theta)$. 
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$y = L(1 - \cos \theta)$
Example: pendulum clock

A pendulum clock that works perfectly on Earth is taken to the Moon.
(a) Does it run fast or slow there? (b) If the clock is started at 12:00 midnight, what will it read after one Earth-day (24.0 h)? Assume that the free-fall acceleration on the Moon is 1.63 m/s².

(a) The period of the pendulum is \( T = 2\pi \sqrt{\frac{L}{g}} \). Thus, on the Moon where the acceleration of gravity is smaller, the period will be longer and the clock will run slow.

(b) The ratio of the pendulum’s period on the Moon to that on Earth is

\[
\frac{T_{\text{Moon}}}{T_{\text{Earth}}} = \frac{2\pi \sqrt{L / g_{\text{Moon}}}}{2\pi \sqrt{L / g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Moon}}}} = \sqrt{\frac{9.80}{1.63}} = 2.45
\]

Hence, the pendulum of the clock on Earth makes 2.45 “ticks” while the clock on the Moon is making 1.00 “tick”. After the Earth clock has ticked off 24.0 h and again reads 12:00 midnight, the Moon clock will have ticked off \( \frac{24.0 \text{ h}}{2.45} = 9.79 \text{ h} \) and will read 9:47 AM.
Physical Pendulum

A **physical pendulum** is any rigid object that is free to oscillate about some fixed axis. The period of oscillation of a physical pendulum is not necessarily the same as that of a simple pendulum.
Damped Oscillations

- Only ideal systems oscillate indefinitely
- In real systems, friction retards the motion
- Friction reduces the total energy of the system and the oscillation is said to be *damped*
Damped Oscillations, cont.

- Damped motion varies depending on the fluid used
  - With a low viscosity fluid, the vibrating motion is preserved, but the amplitude of vibration decreases in time and the motion ultimately ceases
    - This is known as underdamped oscillation
Forced Oscillations and Resonance

A force can be applied periodically to a damped oscillator (a forced oscillation).

When the force is applied at the natural frequency of the system, the amplitude of the oscillations will be a maximum. This condition is called resonance.
Let’s watch a movie!