• Thermal physics
  ➢ Kinetic theory
Lightning Review

Last lecture:

1. Thermal physics
   ✔ Ideal gas. Kinetic theory of gases.

Review Problem 1: Which has more molecules: (1) a mole of nitrogen (N2) gas or (2) a mole of oxygen (O2) gas?

Review Problem 2: Which weights more: (1) a mole of nitrogen (N2) gas or (2) a mole of oxygen (O2) gas?
Kinetic Theory of the Ideal Gas

An **ideal gas** is a dilute gas where the particles act as point particles with no interactions except for elastic collisions.

Gas particles have random motions. Each time a particle collides with the walls of its container there is a force exerted on the wall. The force per unit area on the wall is equal to the pressure in the gas.

The pressure will depend on:

- The number of gas particles
- Frequency of collisions with the walls
- Amount of momentum transferred during each collision
• The pressure is proportional to the number of molecules per unit volume and to the average translational kinetic energy of a molecule

\[ P = \frac{2}{3} \frac{N}{V} \langle K_{tr} \rangle \]

Where \( \langle K_{tr} \rangle \) is the average translational kinetic energy of the gas particles; it depends on the temperature of the gas.

\[ \langle K_{tr} \rangle = \frac{3}{2} kT \]
The average kinetic energy also depends on the rms speed of the gas

\[
\langle K_{\text{tr}} \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m v_{\text{rms}}^2
\]

where the rms speed is

\[
\langle K_{\text{tr}} \rangle = \frac{3}{2} kT = \frac{1}{2} m v_{\text{rms}}^2
\]

\[v_{\text{rms}} = \sqrt{\frac{3kT}{m}}\]
The distribution of speeds in a gas is given by the Maxwell-Boltzmann Distribution.
The distribution of speeds in a gas is given by the **Maxwell-Boltzmann Distribution**.
Example: What is the temperature of an ideal gas whose molecules have an average translational kinetic energy of $3.20 \times 10^{-20}$ J?

Idea: average translational kinetic energy of the gas particles depends on the temperature of the gas:

$$\langle K_{tr} \rangle = \frac{3}{2} kT$$

$$T = \frac{2\langle K_{tr} \rangle}{3k} = 1550 \text{ K}$$
Temperature and Reaction Rates

For a chemical reaction to proceed, the reactants must have a minimum amount of kinetic energy called *activation energy* \((E_a)\).

If \( E_a \gg \frac{3}{2} kT \)

then only molecules in the high speed tail of Maxwell-Boltzmann distribution can react. When this is the situation, the reaction rates are an exponential function of \( T \).

\[
\text{reaction rates} \propto e^{-\left(\frac{E_a}{kT}\right)}
\]
Example: The reaction rate for the hydrolysis of benzoyl-l-arginine amide by trypsin at 10.0 °C is 1.878 times faster than at 5.0 °C. Assuming that the reaction rate is exponential, what is the activation energy?

\[ r_1 \propto e^{-\left(\frac{E_a}{kT_1}\right)} \]

\[ r_2 \propto e^{-\left(\frac{E_a}{kT_2}\right)} \]

where \( T_1 = 10.0 \, ^\circ C = 283 \, K \) and \( T_2 = 5 \, ^\circ C = 278 \, K \); and \( r_1 = 2.878 \, r_2 \).

The ratio of the reaction rates is

\[ \frac{r_1}{r_2} = \exp\left(-\frac{E_a}{kT_1} + \frac{E_a}{kT_2}\right) \]

Solving for the activation energy gives:

\[ E_a = \frac{k \ln\left(\frac{r_1}{r_2}\right)}{\left(\frac{1}{T_2} - \frac{1}{T_1}\right)} = \frac{(1.38 \times 10^{-23} \, J/K)\ln(1.878)}{\left(\frac{1}{278 \, K} - \frac{1}{283 \, K}\right)} = 1.37 \times 10^{-19} \, J \]
Diffusion

On average, a gas particle will be able to travel a distance

\[ \Lambda = \frac{1}{\sqrt{2\pi d^2 \left(\frac{N}{V}\right)}} \]

before colliding with another particle. This is the \textbf{mean free path}. The quantity \( \pi d^2 \) is the cross-sectional area of the particle.
After a collision, the molecules involved will have their direction of travel changed. Successive collisions produce a random walk trajectory.
Substances will move from areas of high concentration to areas of lower concentration. This process is called **diffusion**.

In a time $t$, the rms displacement in one direction is:

$$x_{\text{rms}} = \sqrt{2Dt} \quad \text{D is the diffusion constant}$$
Example: Estimate the time it takes a sucrose molecule to move 5.00 mm in one direction by diffusion in water. Assume there is no current in the water.

Recall that

\[ x_{\text{rms}} = \sqrt{2Dt} \]

Solve for \( t \)

\[
    t = \frac{x_{\text{rms}}^2}{2D} = \frac{(5.00 \times 10^{-3} \text{ m})^2}{2(5.0 \times 10^{-10} \text{ m}^2/\text{s})} = 25000 \text{ s}
\]
Energy in Thermal Processes
Internal Energy vs. Heat

• *Internal Energy*, $U$, is the energy associated with the microscopic components of the system
  - Includes kinetic and potential energy associated with the random translational, rotational and vibrational motion of the atoms or molecules
  - Also includes the intermolecular potential energy
Example: A child of mass 15 kg climbs to the top of a slide that is 1.7 m above a horizontal run that extends for 0.5 m at the base of the slide. After sliding down, the child comes to rest just before reaching the very end of the horizontal portion of the slide. How much internal energy was generated during this process?

\[ U = mgh \]
\[ KE = 0 \]

The change in mechanical energy of the child is
\[ \Delta E = E_f - E_i = -mgh = -250 \text{ J}. \]

This is the increase in internal energy and is distributed between the child, the slide, and the air.
Internal Energy vs. Heat

- **Internal Energy**, U, is the energy associated with the microscopic components of the system
  - Includes kinetic and potential energy associated with the random translational, rotational and vibrational motion of the atoms or molecules
  - Also includes the intermolecular potential energy

- **Heat** is energy transferred between a system and its environment because of a temperature difference between them
  - The letter \(Q\) is used to represent the amount of energy transferred by heat between a system and its environment
Units of Heat

- **Calorie**
  - An historical unit, before the connection between thermodynamics and mechanics was recognized
  - A *calorie* is the amount of energy necessary to raise the temperature of 1 g of water from 14.5° C to 15.5° C.
    - A Calorie (food calorie) is 1000 cal

- **Joule**
  - 1 cal = 4.186 J
  - This is called the *Mechanical Equivalent of Heat*

- **BTU** (US Customary Unit)
  - BTU stands for British Thermal Unit
  - A *BTU* is the amount of energy necessary to raise the temperature of 1 lb of water from 63° F to 64° F

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<thead>
<tr>
<th>Units</th>
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<tbody>
<tr>
<td><strong>SI</strong></td>
<td><strong>Joule (J)</strong></td>
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<tr>
<td><strong>CGS</strong></td>
<td><strong>Calorie (cal)</strong></td>
</tr>
<tr>
<td><strong>US Customary</strong></td>
<td><strong>BTU (btu)</strong></td>
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Heat Capacity

For many substances, under normal circumstances

\[ \Delta T \propto Q. \]

To be exact, \( Q = C \Delta T \) where \( C \) is the heat capacity.
Specific Heat

• Every substance requires a unique amount of energy per unit mass to change the temperature of that substance by 1°C
  • directly proportional to mass (thus, per unit mass)
• The specific heat, $c$, of a substance is a measure of this amount

$$c = \frac{C}{m} = \frac{Q}{m \Delta T}$$

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<td>Joule/kg °C (J/kg °C)</td>
<td>Calorie/g °C (cal/g °C)</td>
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Notes: Heat and Specific Heat

- \( Q = m \ c \ \Delta T \)
  - \( \Delta T \) is always the final temperature minus the initial temperature
  - When the temperature increases, \( \Delta T \) and \( \Delta Q \) are considered to be positive and energy flows into the system
  - When the temperature decreases, \( \Delta T \) and \( \Delta Q \) are considered to be negative and energy flows out of the system
**Example:** How much heat is needed to raise temperature of aluminum by 5°C?

**Given:**

- Mass: \( m = 0.5 \text{ kg} \)
- Temp. \( \Delta T = 5^\circ C \)
- Specific heat:
  \( c_{Al} = 900 \text{ J/kg}^{\circ}C \)

**Find:**

\[ Q = ? \]

Heat is related to mass and temperature by

\[ Q = mc_{Al}\Delta T \]

\[ = (0.5\text{kg})(900\text{ J/kg}^{\circ}C)(+5^{\circ}C) = +2250 \text{ Joules} \]

Thus, energy is flowing into the system! ✔
Example: A 0.400 kg aluminum teakettle contains 2.00 kg of water at 15.0 °C. How much heat is required to raise the temperature of the water (and kettle) to 100 °C?

The heat needed to raise the temperature of the water to \( T_f \) is

\[
Q_w = m_w c_w \Delta T_w = (2 \text{ kg})(4.186 \text{ kJ/kg °C})(85 \text{ °C}) = 712 \text{ kJ}.
\]

The heat needed to raise the temperature of the aluminum to \( T_f \) is

\[
Q_{Al} = m_{Al} c_{Al} \Delta T_{Al} = (0.4 \text{ kg})(0.900 \text{ kJ/kg °C})(85 \text{ °C}) = 30.6 \text{ kJ}.
\]

Then \( Q_{total} = Q_w + Q_{Al} = 732 \text{ kJ} \).
Consequences of Different Specific Heats

- Water has a **high** specific heat compared to **land**
- On a hot day, the air above the land warms faster
- The warmer air flows upward and cooler air moves toward the beach

What happens at night?

\[
c_{Si} = 700 \text{ J/kg}^\circ \text{C} \]
\[
c_{H_2O} = 4186 \text{ J/kg}^\circ \text{C} \]
Question

What happens at night?

1. same
2. opposite
3. nothing
4. none of the above

How to determine specific heat?
Calorimeter

• A technique for determining the specific heat of a substance is called calorimetry

• A calorimeter is a vessel that is a good insulator that allows a thermal equilibrium to be achieved between substances without any energy loss to the environment
Calorimetry

- Analysis performed using a calorimeter
- Conservation of energy applies to the isolated system
- The energy that leaves the warmer substance equals the energy that enters the water
  - $Q_{\text{cold}} = -Q_{\text{hot}}$
  - Negative sign keeps consistency in the sign convention of $\Delta T$
Example: A 0.010-kg piece of unknown metal heated to 100°C and dropped into the bucket containing 0.5 kg of water at 20°C. Determine specific heat of metal if the final temperature of the system is 50°C

Given:

Mass:  
\[ m_1 = 0.010 \text{ kg} \]
\[ m_2 = 0.5 \text{ kg} \]

Specific heat (water):
\[ c_w = 4186 \text{ J/kg°C} \]

Temperatures:
\[ T_1 = 100 \text{ °C} \]
\[ T_2 = 20 \text{ °C} \]
\[ T_f = 50 \text{ °C} \]

Find:

Specific heat = ?

Conservation of energy: heat lost by metal is the same as heat acquired by water:

\[ Q_{water} + Q_{metal} = 0 \]

Solve this equation:

\[
Q_{water} + Q_{metal} = 0 = m_{metal}c_{metal}\Delta T_{metal} + m_{H_2O}c_{H_2O}\Delta T_{H_2O}
\]
\[
= (0.01kg)c_{metal}(50°C - 100°C) + (0.5kg)(4186 \text{ J/kg°C})(50°C - 20°C)
\]
\[
= (-0.5)c_{metal} + 62790J = 0
\]

\[ c_{metal} = 1.25 \times 10^5 \text{ J/kg°C} \]

iron