PHY 2130
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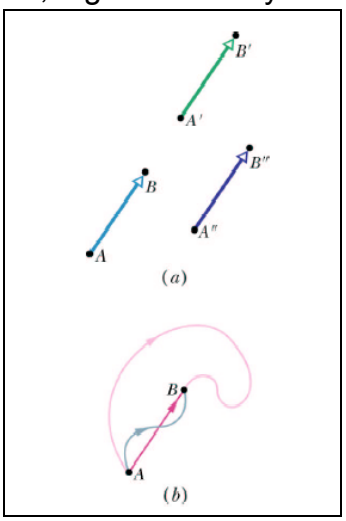
Handout: vectors

A vector has both a magnitude and a direction. A scalar has only a magnitude, no direction. Vectors are indicated by an arrow over the symbol, e.g. the velocity vector is written as $\vec{v}$. Vectors are represented by arrows. Length of arrow = magnitude of vector. The negative of a vector $\vec{b}$ is called $-\vec{b}$ and has the same magnitude as $\vec{b}$ and the opposite direction.



Examples:
Vectors: acceleration, velocity, displacement, force
Scalars: speed, distance traveled, time, potential energy

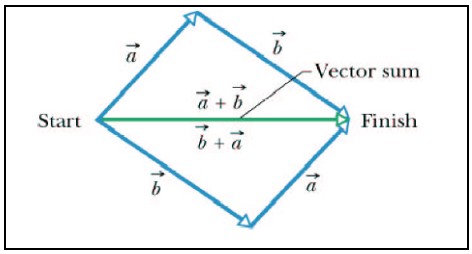
A vector is specified by its magnitude and direction - not by its starting point. The arrows in the picture on the right all represent the same vector!

Adding Vectors

Example: You hike 3 km north from your car, and then change direction and hike 2 km east. How far away from your car are you then?

We need a special procedure to add vectors, so that the directions can be taken into account. There are two of them. First, a graphic method (or "Tail-to-Tip")

1. draw vector $\vec{a}$ with the correct size and angle
2. draw vector $\vec{b}$ with the correct size and angle, so that the tail of $\vec{b}$ starts at the head of $\vec{a}$
3. the sum of vectors $\vec{s} = \vec{a} + \vec{b}$ is then found by connecting the tail of $\vec{a}$ with the head of $\vec{b}$

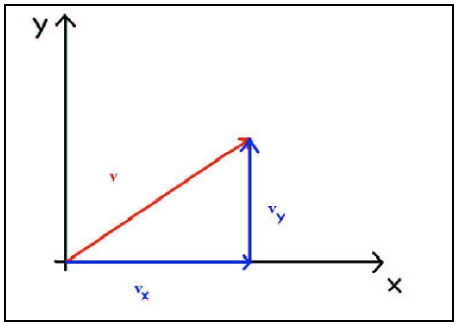


Second method involves computation of components of vectors. Recall that any vector in the xy-plane can be viewed as sum of two components: $\vec{v} = \vec{v}_x + \vec{v}_y$

$\vec{v}_x$: is a vector along the x-axis

$\vec{v}_y$: is a vector along the y-axis

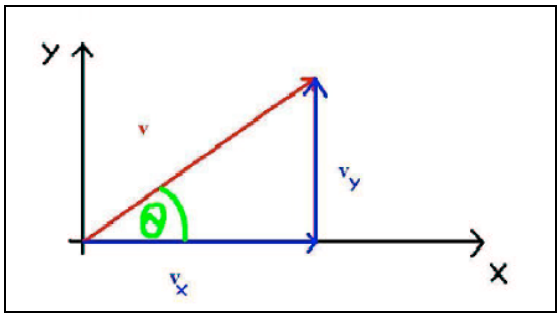
$\vec{v}_x$ and $\vec{v}_y$ form a right triangle



Finding the components, magnitude and direction of a vector using trigonometry

From trigonometry we know that

$\sin\vartheta = \frac{v_y}{v}$, $\cos\vartheta = \frac{v_x}{v}$, so we can find:



1. Components: $v_y = v\sin\vartheta, v_x = v\cos\vartheta$
2. Magnitude: $\left|\vec{v}\right| = \sqrt{v_x^2 + v_y^2}$
3. Direction: $\tan\vartheta = \frac{v_y}{v_x}$

Adding two vectors by components

1. Resolve both vectors into their components (with respect to the same axes), i.e. v_1 with v_{1x}, v_{1y} and v_2 with v_{2x}, v_{2y}
2. Add the x components and the y components separately. This gives you the x and y components of the resultant vector, i.e. $v_{sum,x} = v_{1x} + v_{2x}$ and $v_{sum,y} = v_{1y} + v_{2y}$.
3. Follow instructions above on how to find the magnitude and direction of the resultant vector

Note: never add y and x components!