General Physics (PHY 2140)

Lecture 6

- Electrostatics

- Capacitance and capacitors
  - parallel-plate capacitor
  - capacitors in electric circuits
  - energy stored in a capacitor
  - capacitors with dielectrics

http://www.physics.wayne.edu/~apetrov/PHY2140/

Chapter 16
Last lecture:

1. Potential and potential energy
   ✓ Potential and potential energy of a system of point charges
   ✓ Superposition principle for potential (algebraic sum)
   ✓ Potentials and charged conductors (V is the same in a conductor)
   ✓ Equipotential surfaces (surfaces of constant potential)

2. Capacitance and capacitors

   \[ C = \frac{Q}{\Delta V} \]

Review Problem: A cylindrical piece of insulating material is placed in an external electric field, as shown. The net electric flux passing through the surface of the cylinder is
   a. negative
   b. positive
   c. zero
16.7 The parallel-plate capacitor

The capacitance of a device depends on the geometric arrangement of the conductors:

\[ C = \varepsilon_0 \frac{A}{d} \]

where \( A \) is the area of one of the plates, \( d \) is the separation, \( \varepsilon_0 \) is a constant (permittivity of free space),

\( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \)
Problem: parallel-plate capacitor

A parallel plate capacitor has plates 2.00 m² in area, separated by a distance of 5.00 mm. A potential difference of 10,000 V is applied across the capacitor. Determine the capacitance, the charge on each plate.
A parallel plate capacitor has plates 2.00 m² in area, separated by a distance of 5.00 mm. A potential difference of 10,000 V is applied across the capacitor. Determine the capacitance and the charge on each plate.

Solution:

Given:

- ∆V = 10,000 V
- A = 2.00 m²
- d = 5.00 mm

Find:

- C = ?
- Q = ?

Since we are dealing with the parallel-plate capacitor, the capacitance can be found as

\[ C = \varepsilon_0 \frac{A}{d} = \left( 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \right) \frac{2.00 \, m^2}{5.00 \times 10^{-3} \, m} \]

\[ = 3.54 \times 10^{-9} \, F = 3.54 \, nF \]

Once the capacitance is known, the charge can be found from the definition of a capacitance via charge and potential difference:

\[ Q = C \Delta V = \left( 3.54 \times 10^{-9} \, F \right) \left( 10000 \, V \right) = 3.54 \times 10^{-5} \, C \]
16.8 Combinations of capacitors

- It is very often that more than one capacitor is used in an electric circuit
- We would have to learn how to compute the equivalent capacitance of certain combinations of capacitors
a. Parallel combination

Connecting a battery to the parallel combination of capacitors is equivalent to introducing the same potential difference for both capacitors,

\[ V_1 = V_2 = V \]

A total charge transferred to the system from the battery is the sum of charges of the two capacitors,

\[ Q_1 + Q_2 = Q \]

By definition,

\[ Q_1 = C_1V_1 \quad Q_2 = C_2V_2 \]

Thus, \( C_{eq} \) would be

\[ C_{eq} = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = \frac{Q_1}{V_1} + \frac{Q_2}{V_2} \]

\[ C_{eq} = C_1 + C_2 \]

\[ Q_1 + Q_2 = Q \]

\[ V_1 = V_2 = V \]
Parallel combination: notes

Analogous formula is true for any number of capacitors,

\[ C_{eq} = C_1 + C_2 + C_3 + \ldots \]  
(parallel combination)

It follows that the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors.
Problem: parallel combination of capacitors

A 3 µF capacitor and a 6 µF capacitor are connected in parallel across an 18 V battery. Determine the equivalent capacitance and total charge deposited.
A 3 \( \mu F \) capacitor and a 6 \( \mu F \) capacitor are connected in parallel across an 18 V battery. Determine the equivalent capacitance and total charge deposited.

**Given:**

- \( V = 18 \text{ V} \)
- \( C_1 = 3 \ \mu F \)
- \( C_2 = 6 \ \mu F \)

**Find:**

- \( C_{eq} = ? \)
- \( Q = ? \)

**Diagram:**

First determine equivalent capacitance of \( C_1 \) and \( C_2 \):

\[
C_{12} = C_1 + C_2 = 9 \ \mu F
\]

Next, determine the charge

\[
Q = C\Delta V = (9 \times 10^{-6} \text{ F})(18 \text{ V}) = 1.6 \times 10^{-4} \text{ C}
\]
b. Series combination

Connecting a battery to the serial combination of capacitors is equivalent to introducing the same charge for both capacitors,

\[ Q_1 = Q_2 = Q \]

A voltage induced in the system from the battery is the sum of potential differences across the individual capacitors,

\[ V = V_1 + V_2 \]

By definition,

\[ Q_1 = C_1 V_1 \quad Q_2 = C_2 V_2 \]

Thus, \( C_{eq} \) would be

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ Q_1 = Q_2 = Q \]

\[ V_1 + V_2 = V \]
Series combination: notes

- Analogous formula is true for any number of capacitors,
  \[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \]  
  (series combination)

- It follows that the equivalent capacitance of a series combination of capacitors is always less than any of the individual capacitance in the combination
Problem: series combination of capacitors

A 3 \(\mu\)F capacitor and a 6 \(\mu\)F capacitor are connected in series across an 18 V battery. Determine the equivalent capacitance.
A 3 \( \mu F \) capacitor and a 6 \( \mu F \) capacitor are connected in series across an 18 V battery. Determine the equivalent capacitance and total charge deposited.

**Given:**
- \( V = 18 \) V
- \( C_1 = 3 \) \( \mu F \)
- \( C_2 = 6 \) \( \mu F \)

**Find:**
- \( C_{eq} = ? \)
- \( Q = ? \)

First determine equivalent capacitance of \( C_1 \) and \( C_2 \):

\[
C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 2 \ \mu F
\]

Next, determine the charge

\[
Q = C \Delta V = \left( 2 \times 10^{-6} \ F \right) \left( 18 V \right) = 3.6 \times 10^{-5} C
\]
16.9 Energy stored in a charged capacitor

Consider a battery connected to a capacitor.

A battery must do work to move electrons from one plate to the other. The work done to move a small charge $\Delta q$ across a voltage $V$ is $\Delta W = V \Delta q$.

As the charge increases, $V$ increases so the work to bring $\Delta q$ increases. Using calculus we find that the energy ($U$) stored on a capacitor is given by:

$$U = \frac{1}{2} QV = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$
Example: electric field energy in parallel-plate capacitor

Find electric field energy density (energy per unit volume) in a parallel-plate capacitor

Recall

\[ U = \frac{1}{2} CV^2 \]

\[ C = \frac{\varepsilon_0 A}{d} \quad \text{volume} = Ad \quad V = Ed \]

Thus,

\[ u \equiv \frac{U}{\text{volume}} = \text{energy density} \]

\[ = \frac{1}{2} \frac{\varepsilon_0 A}{d} (Ed)^2 /(Ad) \]

and so, the energy density is

\[ u = \frac{1}{2} \varepsilon_0 E^2 \]
Example: stored energy

In the circuit shown $V = 48\text{V}$, $C_1 = 9\mu\text{F}$, $C_2 = 4\mu\text{F}$ and $C_3 = 8\mu\text{F}$.

(a) determine the equivalent capacitance of the circuit,

(b) determine the energy stored in the combination by calculating the energy stored in the equivalent capacitance.
In the circuit shown \( V = 48 \text{V}, \ C_1 = 9 \mu\text{F}, \ C_2 = 4 \mu\text{F} \) and \( C_3 = 8 \mu\text{F} \).

(a) determine the equivalent capacitance of the circuit, 
(b) determine the energy stored in the combination by calculating the energy stored in the equivalent capacitance,

First determine equivalent capacitance of \( C_2 \) and \( C_3 \):

\[
C_{23} = C_2 + C_3 = 12 \ \mu\text{F}
\]

Next, determine equivalent capacitance of the circuit by noting that \( C_1 \) and \( C_{23} \) are connected in series

\[
C_{eq} \equiv C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = 5.14 \ \mu\text{F}
\]

The energy stored in the capacitor \( C_{123} \) is then

\[
U = \frac{1}{2} CV^2 = \frac{1}{2} \left( 5.14 \times 10^{-6} \text{F} \right) \left( 48 \text{V} \right)^2 = 5.9 \times 10^{-3} \text{J}
\]
16.10 Capacitors with dielectrics

- A dielectrics is an insulating material (rubber, glass, etc.)
- Consider an insolated, charged capacitor

- Notice that the potential difference decreases \((k = V_0/V)\)
- Since charge stayed the same \((Q=Q_0)\) → capacitance increases

\[
C = \frac{Q_0}{V} = \frac{Q_0}{V_0/\kappa} = \frac{\kappa Q_0}{V_0} = \kappa C_0
\]

- dielectric constant: \(k = C/C_0\)
- Dielectric constant is a material property
Capacitors with dielectrics - notes

- Capacitance is multiplied by a factor $k$ when the dielectric fills the region between the plates completely.
- E.g., for a parallel-plate capacitor

$$C = \kappa \varepsilon_0 \frac{A}{d}$$

- The capacitance is limited from above by the electric discharge that can occur through the dielectric material separating the plates.
- In other words, there exists a maximum of the electric field, sometimes called dielectric strength, that can be produced in the dielectric before it breaks down.
# Dielectric constants and dielectric strengths of various materials at room temperature

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric constant, $k$</th>
<th>Dielectric strength (V/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.00</td>
<td>--</td>
</tr>
<tr>
<td>Air</td>
<td>1.00059</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>Water</td>
<td>80</td>
<td>--</td>
</tr>
<tr>
<td>Fused quartz</td>
<td>3.78</td>
<td>$9 \times 10^6$</td>
</tr>
</tbody>
</table>

For a more complete list, see Table 16.1
Example

Take a parallel plate capacitor whose plates have an area of 2000 cm² and are separated by a distance of 1 cm. The capacitor is charged to an initial voltage of 3 kV and then disconnected from the charging source. An insulating material is placed between the plates, completely filling the space, resulting in a decrease in the capacitor's voltage to 1 kV. Determine the original and new capacitance, the charge on the capacitor, and the dielectric constant of the material.
Take a parallel plate capacitor whose plates have an area of 2 m² and are separated by a distance of 1 cm. The capacitor is charged to an initial voltage of 3 kV and then disconnected from the charging source. An insulating material is placed between the plates, completely filling the space, resulting in a decrease in the capacitor's voltage to 1 kV. Determine the original and new capacitance, the charge on the capacitor, and the dielectric constant of the material.

**Given:**

ΔV₁ = 3,000 V  
ΔV₂ = 1,000 V  
A = 2.00 m²  
d = 0.01 m

**Find:**

C = ?  
C₀ = ?  
Q = ?  
k = ?

Since we are dealing with the parallel-plate capacitor, the original capacitance can be found as

\[
C = \varepsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}\right) \frac{2.00 \, m^2}{1.00 \times 10^{-3} \, m} = 18 \, nF
\]

The dielectric constant and the new capacitance are

\[
C = \kappa C_0 = \frac{\Delta V_1}{\Delta V_2} C_0 = 0.33 \cdot 18nF = 6nF
\]

The charge on the capacitor can be found to be

\[
Q = CV = \left(18 \times 10^{-9} \, F\right) \left(3000V\right) = 5.4 \times 10^{-5} \, C
\]
How does an insulating dielectric material reduce electric fields by producing effective surface charge densities?

Reorientation of polar molecules

Induced polarization of non-polar molecules

Dielectric Breakdown: breaking of molecular bonds/ionization of molecules.