Lecture 8

- Electrodynamics
  - Electric current
    - current and drift speed
    - resistance and Ohm’s law
    - resistivity
    - temperature variation of resistance
    - electrical energy and power

http://www.physics.wayne.edu/~apetrov/PHY2140/

Chapter 17
Department of Physics and Astronomy announces the Fall 2003 opening of
**The Physics Resource Center**
on Monday, September 22 in
**Room 172 of Physics Research Building.**

**Hours of operation:**

Mondays, Tuesdays, Wednesdays 11 AM to 6 PM
Thursdays and Fridays 11 AM to 3 PM

Undergraduate students taking PHY2130-2140 will be able to get assistance in this Center with their homework, labwork and other issues related to their physics course.

The Center will be open: Monday, September 22 to Wednesday, December 10, 2003.
Lightning Review

Last lecture:

1. Capacitance and capacitors
   - Capacitors with dielectrics ($C \uparrow$ if $\kappa \uparrow$)

2. Current and resistance
   - Electric current
   - Current and drift speed

Review Problem: A parallel-plate capacitor is attached to a battery that maintains a constant potential difference $V$ between the plates. While the battery is still connected, a glass slab is inserted so as to just fill the space between the plates. The stored energy
   - a. increases
   - b. decreases
   - c. remains the same

$$U = \frac{1}{2} Q V = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$
$$C = \kappa \varepsilon_0 \frac{A}{d}, \quad C = \kappa C_0$$
$$I = \frac{\Delta Q}{\Delta t}$$
$$I = n q v_d A$$
15.2 Current and Drift Speed

Consider the current on a conductor of cross-sectional area $A$. 

\[ I = \frac{q}{\Delta t} \]
15.2 Current and Drift Speed (2)

Volume of an element of length $\Delta x$ is: $\Delta V = A \Delta x$.
Let $n$ be the number of carriers per unit of volume.
The total number of carriers in $\Delta V$ is: $n A \Delta x$.
The charge in this volume is: $\Delta Q = (n A \Delta x)q$.
Distance traveled at drift speed $v_d$ by carrier in time $\Delta t$:
$\Delta x = v_d \Delta t$.
Hence: $\Delta Q = (n A v_d \Delta t)q$.
The current through the conductor:
$$I = \frac{\Delta Q}{\Delta t} = n A v_d q.$$
15.2 Current and Drift Speed (3)

- In an isolated conductor, *charge carriers move randomly in all directions*.
- When an external potential is applied across the conductor, it creates an electric field inside which produces a force on the electron.
- Electrons however still have quite a random path.
- As they travel through the material, electrons collide with other electrons, and nuclei, thereby losing or gaining energy.
- *The work done by the field exceeds the loss by collisions*.
- The electrons then tend to drift preferentially in one direction.
Question:
A copper wire of cross-sectional area $3.00 \times 10^{-6} \text{ m}^2$ carries a current of 10. A. Assuming that each copper atom contributes one free electron to the metal, find the drift speed of the electron in this wire. The density of copper is 8.95 g/cm$^3$. 
Question:
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Reasoning: We know:
• A = 3.00x10^{-6} \text{ m}^2
• I = 10 \text{ A}.
• \rho = 8.95 \text{ g/cm}^3.
• q = 1.6 \times 10^{-19} \text{ C}.
• n = 6.02x10^{23} \text{ atom/mol} \times 8.95 \text{ g/cm}^3 \times (63.5 \text{ g/mol})^{-1}
• n = 8.48 \times 10^{22} \text{ electrons/cm}^3.
Question:
A copper wire of cross-sectional area $3.00 \times 10^{-6} \text{ m}^2$ carries a current of 10 A. Assuming that each copper atom contributes one free electron to the metal, find the drift speed of the electron in this wire. The density of copper is $8.95 \text{ g/cm}^3$.

Ingredients:
$A = 3.00 \times 10^{-6} \text{ m}^2$ ; $I = 10 \text{ A}$.; $\rho = 8.95 \text{ g/cm}^3$.; $q = 1.6 \times 10^{-19} \text{ C}$. 
$n = 8.48 \times 10^{22} \text{ electrons/ cm}^3$.

$$v_d = \frac{I}{nqA} = \left(\frac{10.0 \text{ C/s}}{\left(8.48 \times 10^{22} \text{ electrons/m}^3\right)\left(1.6 \times 10^{-19} \text{ C}\right)\left(3.00 \times 10^{-6} \text{ m}^2\right)}\right)$$

$$= 2.46 \times 10^{-6} \text{ m/s}$$
15.2 Current and Drift Speed - Comments

- Drift speeds are usually **very small**.
- Drift speed much smaller than the average speed between collisions.
  - Electrons traveling at $2.46 \times 10^{-6}$ m/s would take 68 min to travel 1m.
- So why does light turn on so quickly when one flips a switch?
  - The info (electric field) travels at roughly $10^8$ m/s…
Consider a wire has a long conical shape. How does the velocity of the electrons vary along the wire?

Every portion of the wire carries the same current: as the cross sectional area decreases, the drift velocity must increase to carry the same value of current. This is due to the electrical field lines being compressed into a smaller area, thereby increasing the strength of the electric field.
When a voltage (potential difference) is applied across the ends of a metallic conductor, the current is found to be proportional to the applied voltage.

\[ I \propto \Delta V \]
17.3 Definition of Resistance

In situations where the proportionality is exact, one can write:

\[ \Delta V = IR \]

- The proportionality constant \( R \) is called resistance of the conductor.
- The resistance is defined as the ratio.

\[ R = \frac{\Delta V}{I} \]
17.3 Resistance - Units

- In SI, resistance is expressed in volts per ampere.
- A special name is given: ohms (Ω).

Example: if a potential difference of 10 V applied across a conductor produces a 0.2 A current, then one concludes the conductor has a resistance of 10 V/0.2 a = 50 Ω.
17.3 Ohm’s Law

- Resistance in a conductor arises because of collisions between electrons and fixed charges within the material.
- In many materials, including most metals, the resistance is constant over a wide range of applied voltages.
- This is a statement of Ohm’s law.

Georg Simon Ohm
(1787-1854)
Non-Linear or Non-Ohmic Material

Linear or Ohmic Material

Most metals, ceramics

Semiconductors e.g. devices called diodes
Ohm’s Law

\[ \Delta V = IR \]

R understood to be independent of \( \Delta V \).
Resistor: a conductor that provides a specified resistance in an electric circuit.

\[ V = IR \]
Example:
Resistance of a Steam Iron

All household electric devices are required to have a specified resistance (as well as many other characteristics...). Consider that the plate of a certain steam iron states the iron carries a current of 7.40 A when connected to a 120 V source. What is the resistance of the steam iron?

\[
R = \frac{\Delta V}{I} = \frac{120V}{7.40A} = 16.2 \Omega
\]
17.4 Resistivity - Intro

- Electrons moving inside a conductor subject to an external potential constantly collide with atoms of the conductor.
- They lose energy and are repeatedly re-accelerated by the electric field produced by the external potential.
- The collision process is equivalent to an internal friction.
- This is the origin of a material’s resistance.
17.4 Resistivity - Definition

The resistance of an ohmic conductor is proportional to the its length, \( l \), and inversely proportional to the cross section area, \( A \), of the conductor.

\[
R = \rho \frac{l}{A}
\]

• The constant of proportionality \( \rho \) is called the resistivity of the material.
17.4 Resistivity - Remarks

- Every material has a characteristic resistivity that depends on its electronic structure, and the temperature.
- Good **conductors** have low resistivity.
- **Insulators** have high resistivity.

- Analogy to the flow of water through a pipe.
17.4 Resistivity - Units

\[ R = \rho \frac{l}{A} \quad \rho = \frac{RA}{l} \]

- Resistance expressed in Ohms,
- Length in meter,
- Area are m\(^2\),
- Resistivity thus has units of \(\Omega m\).
Resistivity of various materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity (10^{-8} , \Omega m)</th>
<th>Material</th>
<th>Resistivity (10^{-8} , \Omega m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>1.61</td>
<td>Bismuth</td>
<td>106.8</td>
</tr>
<tr>
<td>Copper</td>
<td>1.70</td>
<td>Plutonium</td>
<td>141.4</td>
</tr>
<tr>
<td>Gold</td>
<td>2.20</td>
<td>Graphite</td>
<td>1375</td>
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<tr>
<td>Aluminum</td>
<td>2.65</td>
<td>Germanium</td>
<td>4.6 \times 10^7</td>
</tr>
<tr>
<td>Pure Silicon</td>
<td>3.5</td>
<td>Diamond</td>
<td>2.7 \times 10^9</td>
</tr>
<tr>
<td>Calcium</td>
<td>3.91</td>
<td>Deionized water</td>
<td>1.8 \times 10^{13}</td>
</tr>
<tr>
<td>Sodium</td>
<td>4.75</td>
<td>Iodine</td>
<td>1.3 \times 10^{15}</td>
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<td>Tungsten</td>
<td>5.3</td>
<td>Phosphorus</td>
<td>1 \times 10^{17}</td>
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<tr>
<td>Brass</td>
<td>7.0</td>
<td>Quartz</td>
<td>1 \times 10^{21}</td>
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<tr>
<td>Uranium</td>
<td>30.0</td>
<td>Alumina</td>
<td>1 \times 10^{22}</td>
</tr>
<tr>
<td>Mercury</td>
<td>98.4</td>
<td>Sulfur</td>
<td>2 \times 10^{23}</td>
</tr>
</tbody>
</table>
Mini-quiz

Why do old light bulbs give less light than when new?

Answer:
• The filament of a light bulb, made of tungsten, is kept at high temperature when the light bulb is on.
  • It tends to evaporate, i.e. to become thinner, thus decreasing in radius, and cross sectional area.
  • Its resistance increases with time.
  • The current going though the filament then decreases with time – and so does its luminosity.
• Tungsten atoms evaporate off the filament and end up on the inner surface of the bulb.
  • Over time, the glass becomes less transparent and therefore less luminous.
17.4 Resistivity - Example

(a) Calculate the resistance per unit length of a 22-gauge nichrome wire of radius 0.321 m.

Cross section: \[ A = \pi r^2 = \pi \left(0.321 \times 10^{-3} m\right)^2 = 3.24 \times 10^{-7} m^2 \]

Resistivity (Table): 1.5 x 10–6 Ωm.

Resistance/unit length: \[ \frac{R}{l} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6} \Omega m}{3.24 \times 10^{-7} m^2} = 4.6 \Omega/m \]
17.4 Resistivity - Example

(b) If a potential difference of 10.0 V is maintained across a 1.0-m length of the nichrome wire, what is the current?

\[ I = \frac{\Delta V}{R} = \frac{10.0\text{V}}{4.6\Omega} = 2.2\text{A} \]
17.4 Temperature Variation of Resistance

- Intro

• The resistivity of a metal depends on many (environmental) factors.
• The most important factor is the temperature.
• For most metals, the resistivity increases with increasing temperature.
• The increased resistivity arises because of larger friction caused by the more violent motion of the atoms of the metal.
For most metals, resistivity increases approx. linearly with temperature.

\[ \rho = \rho_o \left[ 1 + \alpha (T - T_o) \right] \]

- \( \rho \) is the resistivity at temperature \( T \) (measured in Celsius).
- \( \rho_o \) is the reference resistivity at the reference temperature \( T_o \) (usually taken to be 20 °C).
- \( \alpha \) is a parameter called temperature coefficient of resistivity.

For a conductor with fixed cross section.

\[ R = R_o \left[ 1 + \alpha (T - T_o) \right] \]
17.5 Temperature Variation of Resistance - Example

Platinum Resistance Thermometer
A resistance thermometer, which measures temperature by measuring the change in the resistance of a conductor, is made of platinum and has a resistance of 50.0 $\Omega$ at 20°C. When the device is immersed in a vessel containing melting indium, its resistance increases to 76.8 $\Omega$. Find the melting point of Indium.

Solution:
Using $\alpha=3.92\times10^{-3}(^{\circ}\text{C})^{-1}$ from table 17.1.
Platinum Resistance Thermometer

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Solution:

Using $\alpha=3.92 \times 10^{-3} \, (^{\circ}C)^{-1}$ from table 17.1.

$R_o = 50.0 \, \Omega$.

$T_o = 20^\circ C$.

$R = 76.8 \, \Omega$.

$$T - T_o = \frac{R - R_o}{\alpha R_o} = \frac{76.8\Omega - 50.0\Omega}{3.92 \times 10^{-3} \, (^{\circ}C)^{-1} \times 50.0\Omega}$$

$$= 137^\circ C$$

$$T = 157^\circ C$$