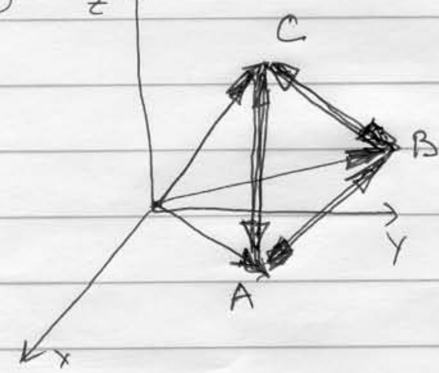
Hw 1

(1)



Show: $\vec{AB} + \vec{BC} + \vec{CA} = 0$

$\vec{AB} = -\vec{A} + \vec{B} = \vec{B} - \vec{A}$,
$\vec{BC} = -\vec{B} + \vec{C} = \vec{C} - \vec{B}$,
$\vec{CA} = -\vec{C} + \vec{A} = \vec{A} - \vec{C}$.

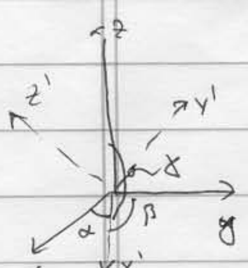
Thus: $\vec{AB} + \vec{BC} + \vec{CA} = \cancel{\vec{B}} - \cancel{\vec{A}} + \cancel{\vec{C}} - \cancel{\vec{B}} + \cancel{\vec{A}} - \cancel{\vec{C}} = 0$

QED.

(2) Show that $\sum_{i=1}^{N} a_{ji} a_{ki} = \delta_{jk}$:

Since $a_{ji} = \frac{\partial x'_j}{\partial x_i}$ & $a_{ki} = \frac{\partial x_i}{\partial x'_k}$:

$$\sum_{i=1}^{N}{}' \frac{\partial x'_j}{\partial x_i} \frac{\partial x_i}{\partial x'_k} = \frac{\partial x'_j}{\partial x'_k} = \underline{\underline{\delta_{jk}}}. \quad \text{QED}.$$

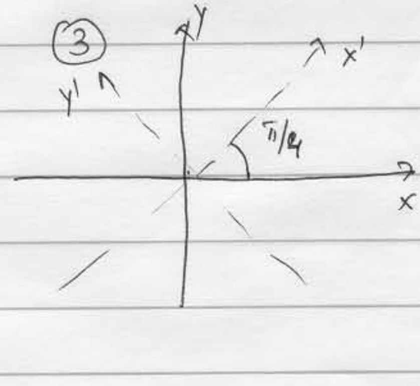


In 3-d: $a_{11} = \cos\alpha$, $a_{12} = \cos\beta$, $a_{13} = \cos\gamma$

$\cos(x'_1, x_1)$ $\cos(x'_1, x_2)$ $\cos(x'_1, x_3)$

Take $j = k = 1 \Rightarrow \delta_{11} = 1 = \sum_{i=1}^{3} a_{1i} a_{1i} = a_{11}^2 + a_{12} a_{12} + a_{13} a_{13}$
$= \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ✓

(3)



$\hat{\vec{x}}' \cdot \hat{\vec{x}}' = 1 = \left(\hat{\vec{x}} \cdot \cos\phi + \hat{\vec{y}} \sin\phi\right) \cdot \left(\hat{\vec{x}} \cos\phi + \hat{\vec{y}} \sin\phi\right) = \left\{ \phi = \frac{\pi}{4} \Rightarrow \cos\phi = \sin\phi = \sqrt{2}/2 \right\}$

$= \frac{1}{2} \left(\underbrace{\hat{\vec{x}} \cdot \hat{\vec{x}}}_{1} + \underbrace{\hat{\vec{y}} \cdot \hat{\vec{y}}}_{1} \right) + \frac{2}{2}\, \hat{\vec{x}} \cdot \hat{\vec{y}} =$

$= 1 + \hat{\vec{x}} \cdot \hat{\vec{y}} = 1 \iff \underline{\hat{\vec{x}} \cdot \hat{\vec{y}} = 0}$, QED