1. Show: \( \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{0} \)
   \[
   \overrightarrow{AB} = -\overrightarrow{A} + \overrightarrow{B} = \overrightarrow{B} - \overrightarrow{A},
   \overrightarrow{BC} = -\overrightarrow{B} + \overrightarrow{C} = \overrightarrow{C} - \overrightarrow{B},
   \overrightarrow{CA} = -\overrightarrow{C} + \overrightarrow{A} = \overrightarrow{A} - \overrightarrow{C}.
   \]

   Thus: \( \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{B} - \overrightarrow{A} + \overrightarrow{C} - \overrightarrow{B} + \overrightarrow{A} - \overrightarrow{C} = \overrightarrow{0} \)
   \( \text{Q.E.D.} \)

2. Show that \( \sum_{i=1}^{N} a_{ij} \cdot a_{ki} = \delta_{jk} \):
   Since \( a_{ij} = \frac{\partial x_i}{\partial x_j} \) and \( a_{ki} = \frac{\partial x_k}{\partial x_i} \),

   \[
   \sum_{i=1}^{N} \frac{\partial x_j}{\partial x_i} \cdot \frac{\partial x_k}{\partial x_i} = \frac{\partial x_j}{\partial x_k} = \delta_{jk}. \text{ Q.E.D.}
   \]

3. In 3rd:
   \( a_{11} = \cos \phi, \ a_{12} = \cos \beta, \ a_{13} = \cos \gamma \)
   \[ \cos \left( x_1^i, x_1^j \right) \cos \left( x_1^j, x_2^j \right) \cos \left( x_1^j, x_3^j \right) \]

   Take \( j = k = 1 \) so \( \delta_{ij} = 1 = \sum_{i=1}^{3} a_{1i} \cdot a_{1i} = a_{11}^2 + a_{12} a_{12} + a_{13} a_{13} = \cos \phi + \cos \beta + \cos \gamma = 1 \)

3. \( 
   \frac{\hat{x}_i \cdot \hat{x}_j}{1} = \left( \hat{x}_i \cos \phi + \hat{y}_i \sin \phi \right) \cdot \left( \hat{x}_j \cos \phi + \hat{y}_j \sin \phi \right) = \left( \frac{x_1 \cdot x_j}{\sqrt{2}} \right) \cos \phi + \left( \frac{y_1 \cdot y_j}{\sqrt{2}} \right) \sin \phi
   \]

   \[ \frac{\hat{x}_i \cdot \hat{x}_j}{2} + \frac{\hat{y}_i \cdot \hat{y}_j}{2} = \frac{1}{2}(x_i \cdot x_j + y_i \cdot y_j) = \left( \frac{1}{2} \left( \frac{1}{2} \hat{x}_i \cdot \hat{x}_j + \frac{1}{2} \hat{y}_i \cdot \hat{y}_j \right) \right) = 1 \]

   \( \hat{x} \cdot \hat{y} = 0 \), \( \text{Q.E.D.} \)