HW 2

① Prove $(\vec{A}\times\vec{B})\cdot(\vec{A}\times\vec{B}) = (AB)^2 - (\vec{A}\cdot\vec{B})^2$.

Let's use Levi-Civitta symbol:

$$\vec{A}\times\vec{B}\Big|_i = \epsilon_{ijk} A_j B_k \text{ , so:}$$

$$(\vec{A}\times\vec{B})\times(\vec{A}\times\vec{B}) = \epsilon_{ijk} A_j B_k \epsilon_{ilm} A_l B_m =$$

$$= \underbrace{\epsilon_{ijk}\epsilon_{ilm}}_{\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}} A_j B_k A_l B_m = A^2B^2 - (\vec{A}\cdot\vec{B})^2$$

QED.

② $\vec{F} = q(\vec{v}\times\vec{B})$

$$\hookrightarrow \frac{\vec{F}}{q} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}(v_y B_z - v_z B_y) + \hat{y}(v_z B_x - v_x B_z) + \hat{z}(v_x B_y - v_y B_x) \text{ ;}$$

a) $\vec{v} = \hat{x} = (1,0,0)$: $2\hat{z} - 4\hat{y} = B_y\hat{z} - B_z\hat{y}$

b) $\vec{v} = \hat{y} = (0,1,0)$: $4\hat{x} - \hat{z} = B_z\hat{x} - B_x\hat{z}$

c) $\vec{v} = \hat{z} = (0,0,1)$: $\hat{y} - 2\hat{x} = -B_y\hat{x} + B_x\hat{y}$

$$\boxed{\begin{array}{l} B_x = 1,\ B_y = 2,\ B_z = 4 \\ \text{or } \vec{B} = \hat{x} + 2\hat{y} + 4\hat{z} \end{array}}$$

③ $\vec{L} = m\vec{r}\times\vec{v} = m\underset{A}{\vec{r}}\times\underset{B}{\vec{\omega}}\times\underset{C}{\vec{r}} = m(\underset{B}{\vec{\omega}}\,\underset{AC}{\vec{r}^2} - \underset{C}{\vec{r}}(\underset{A}{\vec{r}}\cdot\underset{B}{\vec{\omega}}))$ ✓

$$= mr^2(\vec{\omega}\,\underbrace{\cancel{\hat{r}^2}}_{1} - \hat{r}(\hat{r}\cdot\vec{\omega})).$$

If $\vec{r}\cdot\vec{\omega} = 0$, $\boxed{\vec{L} = mr^2\vec{\omega}}$