$\boxed{\text{Hw 3}}$ 1

1. a) unit vector $\perp$ to $x^2+y^2+z^2=3$ @ $(1,1,1)$:

↳ equation of the surface:

$$F(x,y,z) = x^2+y^2+z^2-3=0$$

Normal @ point $(x_0,y_0,z_0)=(1,1,1)$ is given by a gradient:

$$\vec{\nabla}F(x,y,z)\Big|_{x_i=x_{0_i}} = \vec{n}$$

$$\hookrightarrow \underset{\sim}{\vec{n}} = \hat{x}\frac{\partial}{\partial x}F + \hat{y}\frac{\partial}{\partial y}F + \hat{z}\frac{\partial}{\partial z}F\Big|_{x_i=x_{0_i}} =$$

$|\underset{\sim}{\vec{n}}|=\sqrt{3\cdot 2^2}=$
$=\sqrt{12}=\underline{\underline{2\sqrt{3}}}$

$$= \hat{\vec{x}}\cdot 2x\Big|_{x=1} + \hat{\vec{y}}\,2y\Big|_{y=1} + \hat{\vec{z}}\,2z\Big|_{z=1} =$$

$$= 2\hat{\vec{x}} + 2\hat{\vec{y}} + 2\hat{\vec{z}}, \quad \vec{n} = \frac{\vec{\underset{\sim}{n}}}{|\vec{\underset{\sim}{n}}|} = \frac{1}{\sqrt{3}}(\hat{\vec{x}}+\hat{\vec{y}}+\hat{\vec{z}}).$$

b) Derive eqn. of the plane, tangent to the surface at $(1,1,1)$:

Eqn. of the plane: $\vec{r} = \vec{a} + \alpha\vec{b} + \beta\vec{c}$

or $ax+by+cz=d$, (a,b,c) is <u>normal</u>.

$$\hookrightarrow \boxed{a=2,\ b=2,\ c=2}$$

For d: $F(1,1,1)=0 \Leftrightarrow 2x+2y+2z\Big|_{x,y,z=1,1,1} = d$

$$\boxed{d=6}$$

or: $(a,b,c)\perp$ plane;
$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$
1 1 1

$$\hookrightarrow \boxed{2x+2y+2z=6}\ \boxed{x+y+z=3}$$