1. a) unit vector \( \mathbf{n} \) to \( x^2 + y^2 + z^2 = 3 \) at \((1, 1, 1)\):

\[ F(x, y, z) = x^2 + y^2 + z^2 - 3 = 0 \]

Equation of the surface:

\[ \nabla F(x, y, z) = \frac{\partial F}{\partial x} \mathbf{i} + \frac{\partial F}{\partial y} \mathbf{j} + \frac{\partial F}{\partial z} \mathbf{k} \]

Normal @ point \((x_0, y_0, z_0) = (1, 1, 1)\) is given by a gradient:

\[ \mathbf{n} = \nabla F(x, y, z) \bigg|_{x=x_0, y=y_0, z=z_0} \]

\[ = \mathbf{i} \cdot 2x + \mathbf{j} \cdot 2y + \mathbf{k} \cdot 2z \bigg|_{x=1, y=1, z=1} \]

\[ = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \quad \mathbf{n} = \frac{2}{\sqrt{3}} \mathbf{i} + \frac{2}{\sqrt{3}} \mathbf{j} + \frac{2}{\sqrt{3}} \mathbf{k}. \]

6) Derive eqn. of the plane, tangent to the surface at \((1, 1, 1)\):

Eqn. of the plane:

\[ \mathbf{r} = \mathbf{a} + \alpha \mathbf{b} + \beta \mathbf{c} \]

or \( ax + by + cz = d \), \((a, b, c)\) is normal.

\[ a = 2, \ b = 2, \ c = 2 \]

For \( d \):

\[ F(1, 1, 1) = 0 \iff 2x + 2y + 2z = d \]

or: \((a, b, c) \perp \text{plane}\):

\[ a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \]

\[ 2x + 2y + 2z = 6, \quad x + y + z = 3 \]

\[ d = 6 \]