(2.) $\vec{r}_{12} = \hat{\vec{x}}(x_1 - x_2) + \hat{\vec{y}}(y_1 - y_2) + \hat{\vec{z}}(z_1 - z_2)$

$\vec{\nabla}_1 |\vec{r}_{12}| = ?$

$$\vec{\nabla}_1 r_{12} = \vec{\nabla}_1 \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2} =$$

$$= \hat{\vec{x}}_1 \frac{\partial}{\partial x}\sqrt{\dots} + \dots =$$

$$= \hat{\vec{x}}_1 \frac{x_1 - x_2}{|\vec{r}_{12}|} + \hat{\vec{y}}_1 \frac{y_1 - y_2}{|\vec{r}_{12}|} + \hat{\vec{z}}_1 \frac{z_1 - z_2}{|\vec{r}_{12}|} =$$

$$= \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \underline{\hat{\vec{r}}_{12}}, \quad \underline{\underline{QED}}.$$

(3) $$d\vec{F} = \frac{\partial \vec{F}}{\partial x}dx + \frac{\partial \vec{F}}{\partial y}dy + \frac{\partial \vec{F}}{\partial z}dz + \frac{\partial \vec{F}}{\partial t}dt =$$

$$= (d\vec{r}\cdot\vec{\nabla})\vec{F} + \frac{\partial \vec{F}}{\partial t}dt \quad ✓$$

(4) $$\vec{\nabla}(uv) = \hat{\vec{x}}\frac{\partial}{\partial x}(uv) + \dots = \hat{\vec{x}}\, u \frac{\partial v}{\partial x} + \hat{\vec{x}}\, v \frac{\partial u}{\partial x} + \dots =$$

$$= v\vec{\nabla}u + u\vec{\nabla}v. \quad \underline{\underline{QED}}$$

a) $$df = \frac{\partial f}{\partial u}(\vec{\nabla}u)\cdot d\vec{r} + \frac{\partial f}{\partial v}\vec{\nabla}v\cdot d\vec{r} = 0$$

$$\Rightarrow \vec{\nabla}u = -\frac{\partial f/\partial v}{\partial f/\partial u}\vec{\nabla}v$$ ← i.e. $\vec{\nabla}u$ & $\vec{\nabla}v$ are paral. in one direction!

$$\Rightarrow \boxed{\vec{\nabla}u \times \vec{\nabla}v = 0 \Leftrightarrow f(u,v) = 0}$$

in 2d!

b) $$(\vec{\nabla}u)\times(\vec{\nabla}v) = \begin{vmatrix} \hat{\vec{x}} & \hat{\vec{y}} & \hat{\vec{z}} \\ \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & 0 \end{vmatrix} = \hat{\vec{z}} \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \partial v/\partial y \end{vmatrix} =$$

$$= \hat{\vec{z}}\, J(u,v/x,y).$$ If $J = 0$ $\Rightarrow$ x... $= 0$