HW 4

(1) $\vec{r} = \hat{x}\, r\cos\omega t + \hat{y}\, r\sin\omega t$; $\vec{r}\,' = -\hat{x}\,\omega r\sin\omega t + \hat{y}\,\omega r\cos\omega t$

(a) $\vec{r}\times\vec{r}\,' = \underbrace{\hat{x}\times\hat{y}}_{\hat{z}}\, r^2\omega\cos^2\omega t + \underbrace{\hat{y}\times\hat{x}}_{-\hat{z}}(-1)\,\omega r^2\sin^2\omega t =$

$= \hat{z}\,\omega r^2(\cos^2\omega t + \sin^2\omega t) = \underline{\underline{\hat{z}\,\omega r^2}}$ ✓

(b) $\vec{r}\,'' = -\hat{x}\,\omega^2 r\cos\omega t - \hat{y}\,\omega^2 r\sin\omega t =$

$= -\omega^2(\underbrace{\hat{x}\, r\cos\omega t + \hat{y}\, r\sin\omega t}_{\vec{r}}) = \underline{\underline{-\omega^2\vec{r}}}$ ✓

(2) $\vec{\nabla}\,(\vec{A}\cdot\vec{B}\times\vec{r}) = \vec{\nabla}\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ x & y & z \end{vmatrix} =$

$= \vec{\nabla}\Big(z(A_xB_y - A_yB_x) + y(-A_xB_z + A_zB_x) + x(A_yB_z - A_zB_y)\Big) =$

$= \hat{x}(A_yB_z - A_zB_y) + \hat{y}(A_zB_x - A_xB_z) + \hat{z}(A_xB_y - A_yB_x) =$

$\equiv \vec{A}\times\vec{B}$. QED

(3) $\psi(\vec{r}) = \dfrac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3}$; $\vec{E} = -\vec{\nabla}\psi$ or

$$\vec{E} = -\frac{1}{4\pi\epsilon_0}\vec{\nabla}\left[\frac{\vec{p}\cdot\vec{r}}{r^3}\right] =$$

$$= -\frac{1}{4\pi\epsilon_0}\left[\hat{x}\frac{\partial}{\partial x}\left[\frac{\vec{p}\cdot\vec{r}}{r^3}\right] + \hat{y}\frac{\partial}{\partial y}\left[\frac{\vec{p}\cdot\vec{r}}{r^3}\right] + \hat{z}\frac{\partial}{\partial z}\left[\frac{\vec{p}\cdot\vec{r}}{r^3}\right]\right] =$$

$$\hat{x}\frac{\partial}{\partial x}\left[\frac{p_xx + p_yy + p_zz}{(\sqrt{x^2+y^2+z^2})^3}\right] = -\hat{x}\frac{3x(p_xx + p_yy + p_zz)}{(x^2+y^2+z^2)^{5/2}} + \hat{x}\frac{p_x}{r^3} =$$

(for $\vec{r}\neq 0$)

$$= -\hat{x}\frac{3x(\vec{p}\cdot\vec{r})}{r^5} + \hat{x}\frac{p_x}{r^3}$$

Thus: $\vec{E} = +\dfrac{1}{4\pi\epsilon_0}\left[\hat{x}\left(\dfrac{3x(\vec{p}\cdot\vec{r})}{r^5} - \dfrac{p_x}{r^3}\right) + \hat{y}\left(\dfrac{3y(\vec{p}\cdot\vec{r})}{r^5} - \dfrac{p_y}{r^3}\right) + \ldots\right] =$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3\hat{r}(\vec{p}\cdot\hat{r})}{r^3} - \frac{\vec{p}}{r^3}\right] = \frac{1}{4\pi\epsilon_0}\frac{3\hat{r}(\vec{p}\cdot\hat{r}) - \vec{p}}{r^3},$$