$\boxed{HW5}$

(1) $\boxed{A \& W, 1.9.3}$. Prove that $\vec\nabla \times (\phi \vec\nabla \phi) = 0$

$$\vec\nabla \times (\phi \vec\nabla \phi) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \phi \frac{\partial}{\partial x}\phi & \phi \frac{\partial}{\partial y}\phi & \phi \frac{\partial}{\partial z}\phi \end{vmatrix} = \left\{ \begin{matrix} \text{each component} \\ \text{must be zero} \end{matrix} \right\} =$$

$$= \hat{x}\left(\underbrace{\frac{\partial}{\partial y}\left(\phi \frac{\partial}{\partial z}\phi\right)} - \frac{\partial}{\partial z}\left(\phi \frac{\partial}{\partial y}\phi\right)\right) + \hat{y}(\ldots) + \hat{z}(\ldots) = \underline{0}.$$

$$\left(\frac{\partial}{\partial y}\phi\right)\left(\frac{\partial}{\partial z}\phi\right) + \cancel{\phi \frac{\partial^2}{\partial y \partial z}\phi} - \left(\frac{\partial \phi}{\partial z}\right)\left(\frac{\partial \phi}{\partial y}\right) - \cancel{\phi \frac{\partial^2}{\partial y \partial z}\phi} = 0$$

(similar for $\hat{y}$ & $\hat{z}$).

QED.

(2) $\boxed{A \& W\ 1.9.8}$ if $\vec\nabla^2 \phi = 0$: then $\vec\nabla\phi$ is both irrot. & solen.

let $\vec{V} = \vec\nabla\phi$. $\quad \vec\nabla \cdot \vec\nabla \phi = \vec\nabla \vec{V} = \vec\nabla^2 \phi = 0 \quad \checkmark$

$$\vec\nabla \times \vec\nabla\phi = \vec\nabla \times \vec{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{vmatrix} = \underline{\underline{0}} \quad \checkmark$$

(exchange $\frac{\partial}{\partial x}\frac{\partial}{\partial y}$)

QED.

(3) $\boxed{A \& W\ 1.9.9}$ Show that $(\vec{r} \times \vec\nabla) \cdot (\vec{r} \times \vec\nabla)\psi =$

$$= \underline{r^2 \nabla^2 \psi - r^2 \frac{\partial^2 \psi}{\partial r^2} - 2r \frac{\partial \psi}{\partial r}}.$$

$$(\vec{r} \times \vec\nabla)\psi = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = \left[\hat{x}\left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y}\right) + \hat{y}\left(-x \frac{\partial}{\partial z} + z \frac{\partial}{\partial x}\right) + \hat{z}\left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}\right) \right]\psi$$

Thus:

$$\left[\left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y}\right)\left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y}\right) + \left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z}\right)\left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z}\right) + \right.$$

$$\left. + \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}\right)\left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}\right) \right]\psi =$$

$$= y^2 \frac{\partial^2}{\partial z^2}\psi - y \frac{\partial \psi}{\partial y} - yz \frac{\partial^2 \psi}{\partial z \partial y} - z \frac{\partial \psi}{\partial z} - zy \frac{\partial^2 \psi}{\partial y \partial z} + z^2 \frac{\partial^2 \psi}{\partial y^2} +$$

$$z^2 \frac{\partial^2 \psi}{\partial x^2} - z \frac{\partial \psi}{\partial z} - zx \frac{\partial^2 \psi}{\partial z \partial x} - x \frac{\partial \psi}{\partial x} - xz \frac{\partial^2 \psi}{\partial z \partial x} + x^2 \frac{\partial^2 \psi}{\partial z^2} +$$

$$x^2 \frac{\partial^2 \psi}{\partial y^2} - x \frac{\partial \psi}{\partial x} - xy \frac{\partial^2 \psi}{\partial x \partial y} - y \frac{\partial \psi}{\partial y} - xy \frac{\partial^2 \psi}{\partial x \partial y} + y^2 \frac{\partial^2 \psi}{\partial x^2} =$$