$\boxed{HW6}$

① $\boxed{A \,\&\, W\ 1.11.8}$ Show that $\int_V \vec{J}\, d\tau = 0$ if $\vec{J}\big|_{S,\, S\to\infty} = 0$ &
$\vec{\nabla}\cdot\vec{J} = 0$

<u>Note that:</u> $J_x = \vec{\nabla}\cdot(x\vec{J})$ (same for y, z)

Indeed, $\vec{\nabla}\cdot(x\vec{J}) = (\vec{\nabla}x)\cdot\vec{J} + x\underbrace{\vec{\nabla}\cdot\vec{J}}_{0} = \vec{J}\cdot\hat{x} = J_x$ ✓

$\vec{\nabla}x = \hat{x}\frac{\partial x}{\partial x} + \hat{y}\frac{\partial x}{\partial y} + \hat{z}\frac{\partial x}{\partial z} = \hat{x}$. same for y & z.

Use Gauss' thm: $\int_V \vec{\nabla}\cdot\vec{V}\, d\tau = \oint_{S=\partial V} \vec{V}\cdot d\vec{\sigma}$

& select $\vec{V} = x\vec{J}$ (also $y\vec{J}$ & $z\vec{J}$).

<u>Then:</u> $\int_V \vec{\nabla}\cdot(x\vec{J})\, d\tau = \oint_{S=\partial V} x\,\vec{J}\cdot d\vec{\sigma} = \oint_S x\, \underbrace{J\big|_S}_{=0}\, d\sigma = 0$

same for $y\vec{J}$ & $z\vec{J}$!

<u>Thus:</u> $\int_V \vec{\nabla}(x\vec{J})d\tau \cdot \hat{x} + \int_V \vec{\nabla}(y\vec{J})d\tau\cdot\hat{y} +$

$+ \int_V \vec{\nabla}(z\vec{J})d\tau\cdot\hat{z} = 0 = \int_V \vec{J}\, d\tau = 0$ ✓

QED

as $\vec{J} = J_x\hat{x} + J_y\cdot\hat{y} + J_z\cdot\hat{z}$

② Calculate $\oint \vec{r}\times d\vec{r}$ (A & W 1.12.2)

<u>Note:</u> $\vec{r}\times d\vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & z \\ dx & dy & dz \end{vmatrix} =$

$= \hat{x}(y\,dz - z\,dy) + \hat{y}(-x\,dz + z\,dx) + \hat{z}(x\,dy - y\,dx)$

The loop is in (x,y) plane $\Rightarrow$ only $\hat{z}$-component matters.