PHY5100. Homework 2

This homework assignment is due on **September 24** by 5 pm.

Suggested reading:

G. Arfken and H. Weber, *Mathematical Methods*, Chapter 1.

Problem 1: Some loose ends (A&W, 1.4.7)

Prove that

$$\left(\vec{A} \times \vec{B}\right) \cdot \left(\vec{A} \times \vec{B}\right) = (AB)^2 - \left(\vec{A} \cdot \vec{B}\right)^2 . \tag{1}$$

(we used this relation in class to find the magnitude of a vector product).

Problem 2: Interpreting experimental data (A&W, 1.4.16)

Recall that the magnetic induction $\vec{B}$ is defined by the Lorentz force equation,

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right), \tag{2}$$

where q is electric charge. Carrying out three experiments, we find that

$$\begin{aligned} \vec{v} = \hat{\vec{x}}: \qquad & \frac{\vec{F}}{q} = 3\hat{\vec{z}} - 4\hat{\vec{y}}, \\ \vec{v} = \hat{\vec{y}}: \qquad & \frac{\vec{F}}{q} = 4\hat{\vec{x}} - \hat{\vec{z}}, \\ \vec{v} = \hat{\vec{z}}: \qquad & \frac{\vec{F}}{q} = \hat{\vec{y}} - 2\hat{\vec{x}}. \end{aligned} \tag{3}$$

From these results of these three separate experiments calculate the magnetic induction $\vec{B}$.

Problem 3: Fun with rotations II. (A&W, 1.5.5)

The orbital angular momentum $\vec{L}$ of a particle is given by $\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$, where $\vec{p}$ is a linear momentum. With linear and angular velocity related by $\vec{v} = \vec{\omega} \times \vec{r}$, show that

$$\vec{L} = mr^2 \left[\vec{\omega} - \hat{\vec{r}}\left(\hat{\vec{r}} \cdot \vec{\omega}\right)\right]. \tag{4}$$

Here $\hat{\vec{r}}$ is a unit vector in the $\vec{r}$-direction. What do you get if $\vec{r} \cdot \vec{\omega} = 0$?