PHY5100. Homework 3

This homework assignment is due on **October 1** by 5 pm.

Suggested reading:

G. Arfken and H. Weber, *Mathematical Methods*, Chapter 1.

Problem 1: a sphere (A&W, 1.6.2)

(a) Find a unit vector perpendicular to the surface

$$x^2 + y^2 + z^2 = 3 \tag{1}$$

at the point $(1, 1, 1)$. Lengths are in centimeters.
(b) Derive the equation of the plane, tangent to the surface at $(1, 1, 1)$

Problem 2: fun with a gradient (A&W, 1.6.3)

Given a vector $\vec{r}_{12} = \hat{\vec{x}}(x_1 - x_2) + \hat{\vec{y}}(y_1 - y_2) + \hat{\vec{z}}(z_1 - z_2)$, show that $\vec{\nabla}_1 \cdot \vec{r}_{12}$ (gradient with respect to x_1, y_1, and z_1 of the magniture r_{12}) is a unit vector in the direction of $\vec{r}_{12}$.

Problem 3: Full derivative (A&W, 1.6.4)

If a vector function $\vec{F}$ depends on both space coordinates (x, y, z) and time t, show that

$$d\vec{F} = \left(d\vec{r} \cdot \vec{\nabla}\right) \vec{F} + \frac{\partial \vec{F}}{\partial t} dt. \tag{2}$$

Problem 4: fun with scalar functions(A&W, 1.6.5)

Show that $\vec{\nabla}(uv) = v\vec{\nabla}u + u\vec{\nabla}v$, where u and v are differentiable scalar functions of x, y , and z.

(a) Show that a necessary and sufficient condition that $u(x, y, z)$ and $v(x, y, z)$ are related by some function $f(u, v) = 0$ is that $(\vec{\nabla}u) \times (\vec{\nabla}v) = 0$

(b) If $u = u(x, y)$ and $v = v(x, y)$, show that the condition $(\vec{\nabla} u) \times (\vec{\nabla} v) = 0$ leads to the two-dimensional Jacobian

$$J\left(\frac{u, v}{x, y}\right) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = 0. \tag{3}$$

The functions u and v are assumed differentiable.