PHY5100. Homework 3

This homework assignment is due on **October 10** by 5 pm.

Suggested reading:

G. Arfken and H. Weber, *Mathematical Methods*, Chapter 1.

Problem 1: rotating particle (A&W, 1.7.1)

For a particle moving in a circular orbit $\vec{r} = \hat{\vec{x}}\, r\cos\omega t + \hat{\vec{y}}\, r\sin\omega t$

(a) evaluate $\vec{r} \times \vec{r'}$, with $\vec{r'} = d\vec{r}/dt = \vec{v}$.
(b) Show that $\vec{r''} + \omega^2 \vec{r} = 0$ with $\vec{r''} = d\vec{v}/dt$.

Problem 2: some math (A&W, 1.8.14)

If $\vec{A}$ and $\vec{B}$ are constant vectors, show that

$$\vec{\nabla}\left(\vec{A}\cdot\vec{B}\times\vec{r}\right) = \vec{A}\times\vec{B}. \tag{1}$$

Problem 3: a dipole (A&W, 1.8.16)

An electric dipole of moment $\vec{p}$ is located at the origin. The dipole creates an electric potential at $\vec{r}$ given by

$$\psi\left(\vec{r}\right) = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3}. \tag{2}$$

Find the electric field, $\vec{E} = -\vec{\nabla}\psi$ at $\vec{r}$.

Problem 4: Cauchy-Riemann conditions (A&W, 1.8.18)

The velocity of a two-dimensional flow of liquid is given by

$$\vec{V} = \hat{\vec{x}}\, u(x,y) - \hat{\vec{y}}\, v(x,y). \tag{3}$$

If the liquid is incompressible and the flow is irrotational, show that

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \text{and} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \tag{4}$$

These are the Cauchy-Riemann conditions, which play a big role in complex analysis.