# PHY7400. Homework 3

This homework assignment is due on **October 12**.

## Suggested reading:

Eugen Merzbacher, *Quantum Mechanics*, Chapters 5-6;
L. Landau, E. Lifschits, *Quantum Mechanics*, Chapters 1-3.

## Problem 1: Anticommuting operators (10 pt).

Two operators, $\hat{A}$ and $\hat{B}$, share the same eigenfunction $\psi_{ab}$.

1. Prove that at least one eigenvalue, $a$ (for $\hat{A}$) or $b$ (for $\hat{B}$), is zero, if the operators $\hat{A}$ and $\hat{B}$ *anticommute*, i.e. $\hat{A}\hat{B} + \hat{B}\hat{A} \equiv \{\hat{A}, \hat{B}\} = 0$.
2. Consider a particular case of $\hat{A} = \hat{x}$ and $\hat{B} = \hat{P}$ (with $\hat{P}\psi(x) = \psi(-x)$). Find the eigenfunction $\psi_{xP}(x)$.

## Problem 2: 1-dim motion: first encounter (10 pt).

As you know from classical physics, the Hamiltonian for a single particle of the mass $m$ moving in the potential $V(x)$ is $H = p^2/(2m) + V(x)$. We already know the form of both momentum and coordinate operators in quantum mechanics. It is therefore not difficult to convince oneself that the equation that yields energy eigenvalues is

$$\hat{H}\psi_E(x) = E\psi_E(x), \qquad \hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x). \tag{1}$$

1. Find the eigenvalues of the Hamiltonian (energy levels) and normalized eigenfunctions of discrete spectrum for the delta-function potential, $V(x) = -\alpha\delta(x)$.

2. Find expectation values of kinetic and potential energies for those eigenfunctions.

3. Compute the product of uncertainties of coordinate and momentum for those states and check the uncertainty relation.

## Problem 3: Eigenvalues and eigenfunctions (10 pt).

Find eigenvalues and normalized eigenfunctions of the following Hermitian operator

$$\hat{F} = \alpha\hat{p} + \beta\hat{x}. \tag{2}$$