PHY7400. Homework 4

This homework assignment is due on **October 21**.

Suggested reading:

Eugen Merzbacher, *Quantum Mechanics*, Chapters 5-6.

Problem 1: One δ-function, two δ-functions... (10 pt).

Delta-function potential is a useful approximation in quantum mechanics. Let us discuss some of its implications.

1. Find reflection and transmission coefficients for the potential $U = \alpha\delta(x)$. What happens to reflection coefficient $R(E)$ as $E \to \infty$?
2. Consider a particle incident on the following potential $U = \alpha(\delta(x) + \delta(x-a))$ for $\alpha > 0$ (a "comb" of two delta-functions). Find the energies of this particle at which the reflection coefficient is zero.

Problem 2: Normalization and completeness (10 pt).

Consider particle moving in the "infinite step" potential,

$$U(x) = \begin{cases} 0, & \text{for } x \geq 0 \\ \infty, & \text{for } x < 0 \end{cases} \tag{1}$$

1. Find the normalized wave functions of the stationary states. Use the δ-function normalization condition $\int dx\ \psi^*(E', x)\psi(E, x) = \delta(E' - E)$.
2. Show that the resulting eigensystem forms a complete set of functions. In other words, show that the completeness relation $\int dE\ \psi^*(E, x')\psi(E, x') = \delta(x' - x)$ is satisfied.

Problem 3: Newton's mechanics? (10 pt).

Consider a particle moving in one-dimensional potential $U(x)$. In general, its motion is described by a time-dependent Schrodinger equation for $\psi(x, t)$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi. \tag{2}$$

1. Write a corresponding equation for $\psi^*(x,t)$ and find an expression for the time derivative of the expectation value of momentum operator $\hat{p}$ in state $\psi(x,t)$. In other words, determine

$$\frac{\partial}{\partial t}\langle \hat{p} \rangle.$$

2. Use Shrodinger equation from Eq. (2) to rewrite the previous equation as an equation for expectation values

$$\frac{\partial}{\partial t}\langle \hat{p} \rangle = \langle \hat{O} \rangle, \tag{3}$$

   and thus determine the operator $\hat{O}$. What classical equation of motion does Eq. (3) correspond to?