Physics 7410. Midterm Exam
Spring 2006

Problem 1.

Calculate the first order shift in the ground state of the hydrogen atom caused by the finite size of the proton. Assume the proton is a uniformly charged sphere of radius $r_0 = 10^{-13}$ cm and of charge density ρ_0. To accomplish this

1. Determine ρ_0 from the charge of the proton ($+e$) and its "volume."
2. Use Gauss' law to compute the potential energy *inside* the proton (charged sphere of radius r_0). In other words, recall that

$$U(r) = -e\phi(r) = -e\left(\phi(r_0) + \int_r^{r_0} dr' E(r')\right),$$

   and that $E = q_{inside}/(4\pi\epsilon_0 r^2) = kq_{inside}/r^2$. Also, q_{inside} is determined by the charge confined inside the sphere of radius r.
3. Keeping in mind that the potential for the point-like proton is $U(r) = -ke^2/r$, find the perturbation potential. Apply perturbation theory to compute the required energy shift. Also, use the fact that $r_0 \ll a_0$ (and thus all $r < a_0$ too!) to simplify your calculation of integrals.
4. Recalling that the binding energy of unperturbed ground state is $|E_0| = e^2/(2a_0(4\pi\epsilon_0)) = 13.6$ eV, determine the size of the effect.

Hint: you might find the following information useful. The ground state wave function of the hydrogen atom is

$$\psi_{100}(r, \theta, \phi) = \sqrt{\frac{1}{\pi a_0^3}}e^{-\frac{r}{a_0}},$$

where a_0 is the Bohr radius $a_0 = 0.53 \times 10^{-10}$ m.

Problem 2.

The Hamiltonian describing the interaction of a proton and a neutron in a certain experiment is

$$\hat{H}_0 = J\hat{\vec{S}}_p \cdot \hat{\vec{S}}_n, \tag{1}$$

where J is a coupling constant.

1. What are the eigenvalues and eigenstates of $\hat{H}_0$? For the eigenstates, write your answer in terms of single paritcle $\hat{S}_z$ eigenstates like $|\uparrow_p\downarrow_n\rangle$.
2. At $t = 0$ the system is in its ground state, and a weak external time-dependent magnetic field B_0 in the z direction is turned on such that the interaction acquires a term

   $$\hat{H}' = -(B_0 \cos \omega t)(g_p \hat{S}_{pz} + g_n \hat{S}_{nz}).$$

   Calculate the probability for the system to make a transition to the upper state after time T.

Note: you can use a final formula for periodic perturbation derived in class,

$$P_{t \to f} = \frac{|\langle f|V_0|i\rangle|^2}{\hbar^2} \frac{\sin^2((\omega - \omega_0)T/2)}{(\omega - \omega_0)^2},$$

with $\hbar\omega_0 = E_f - E_i$. Here V_0 is the time-independent part of the potential $\hat{H}'$. In other words, you only have to compute the spin-dependent transition $\langle f|V_0|i\rangle$.

Hint: to compute the relevant spin-dependent matrix elements, find the state vector resulting from the action on the lower energy state vector with the operator $(g_p \hat{S}_{pz} + g_n \hat{S}_{nz})$ and use orthonormality.