

PHY7410. Homework 2

This homework assignment is due on **February 3**.

Suggested reading:

L. Landau and E. Lifshitz, *Quantum Mechanics*, Chapter 6.

Problem 1: Perturbations of boundary conditions.

Consider a quantum particle confined inside a spheroid (an ellipsoid with two equal semi-axes), i.e. in the potential

$$U(x, y, z) = \begin{cases} 0, & \text{for } (x^2 + y^2)/a^2 + z^2/b^2 < 1, \\ \infty, & \text{for } (x^2 + y^2)/a^2 + z^2/b^2 \geq 1, \end{cases} \quad (1)$$

1. Compute exact energy spectrum for the s -wave ($l = m = 0$) states in the particular case of spherical well, i.e. for $a = b = R$. Hint: use polar coordinates and explore the similarity of the obtained radial equation and Schrodinger equation for one-dimensional infinite square well.
2. Using perturbative methods developed in class, compute the ground state energy of the particle in the spheroidal well with $a \neq b$: $|a - b| \ll a$ by evaluating first order correction to the ground state energy of the spherical well considered above.

Problem 2: Variational calculations.

Use variational method to determine ground state energy for the particle confined in the potential

$$U(x) = \begin{cases} Fx, & \text{for } x \geq 0, \\ \infty, & \text{for } x < 0, \end{cases} \quad (2)$$

1. Use trial wave function of the form

$$\psi(x, \alpha) = Ax \exp(-\alpha x)$$

and compare it to the ground state energy calculated exactly.

2. Repeat the calculation for the trial wave function

$$\psi(x, \alpha) = Bx \exp(-\alpha x^2/2).$$

Which wave function gives a better estimate? In both cases show that the virial theorem is satisfied.