PHY7410. Homework 4

This homework assignment is due on **March 1**.

Suggested reading:

L. Landau and E. Lifshitz, *Quantum Mechanics*, Chapter XVII.

Problem 1: Spin dynamics

A very convenient formalism to deal with time-development of a quantum system involves the so-called *time evolution operator*. It can be obtained by formal integration of time-dependent Schrödinger equation,

$$i\hbar \frac{\partial}{\partial t}\psi = \hat{H}\psi \quad \Rightarrow \quad \psi(t) = \exp\left(-i\hat{H}t/\hbar\right)\psi(0) \equiv \hat{U}(t)\psi(0) \tag{1}$$

Let us use this formalism to study spin dynamics.

Suppose that the spin variable of a one-half particle evolves under the influence of the Hamiltonian,

$$\hat{H} = \lambda\hbar\sigma_x, \tag{2}$$

where λ is a real-valued positive constant. Initially, the system is prepared in the spin-up $\hat{S}_z$-eigenstate. In other words, at $t = 0$ the system is at

$$\psi(t = 0) = \chi_\uparrow^{(z)}.$$

1. The spin variable S_z is measured at time t. Use the time evolution operator $U(t)$ to determine the probability that the spin points *down* in the following two different ways:
   (a) First obtain an expression in closed form of $U(t)$ and then answer the question.
   (b) Alternatively, first expand the initial spin wave function $\chi_\uparrow^{(z)}$ in terms of eigenstates of S_x and then answer the question.
2. Another observer measures S_z at intermediate time $t/2$ and then you again measure S_z at time t. Detrmine the probability that you find spin *down* for each of the following cases:
   (a) the other observer obtains spin-up,
   (b) the other observer obtains spin-down,
   (c) you are not told the findings of the other observer.

Problem 2: Clebsh and Gordon

The angular momentum of each of two particles is measured and found to be $j_1 = 3/2$ and $j_2 = 1/2$ respectively, in units of $\hbar$.

1. Following what we did in class, i.e. staring with the state of highest weight, use ladder techniques to construct all possible $|j, m\rangle$ states in terms of the "product basis" $|j_1, m_1\rangle \otimes |j_2, m_2\rangle$. Note: it might be useful to first exhibit the effect of lowering upon the individual $|j, m\rangle$ states, i.e. $|2, m\rangle$, $|3/2, m\rangle$, etc.
2. A measurement of the total angular momentum and its z-component is made, and the results $j = 1, m = 0$ are found. Immediately thereafter, a measurement of the z-component of the angular momenta of paricle 1 and particle 2 individually is made. What are the possible results and with what probability do they occur?

Problem 3: Spin-1 states

Acting with $\hat{S}^2$ and $\hat{S}_\pm$ on the eigenstates of S_z-operator, construct the spin matrices (S_x, S_y, and S_z) for a particle of *spin 1*. Hint: how many eigenstates are there? If in trouble, consult section 4.4.1 of Griffiths' textbook (or get a copy of that section from me).