PHY7410. Homework 5

This homework assignment is due on **March 10**.

Suggested reading:

L. Landau and E. Lifshitz, *Quantum Mechanics*, Chapter XVII (sections 123-128, 130-132).

Problem 1: Born and Born again

A particle of mass m undergoes scattering from the Gaussian proential

$$V(r) = V_0 e^{-\alpha^2 r^2}, \tag{1}$$

where α is a constant with the dimension of inverse length.

1. Determine the Born amplitude $f^B(q^2)$. In your answer, group V_0 with $\hbar$ and m such as to form a dimensionless quantity.
2. Determine the total Born cross section $\sigma^B(k^2)$.
3. Repeat the calculation for the square well potential,

$$V(r) = \begin{cases} V_0, & \text{for } r < a \\ 0, & \text{for } r > a. \end{cases} \tag{2}$$

   Determine the total Born cross section $\sigma^B(0)$ to leading order as $k \to 0$ by first evaluating f^B in that limit.

Problem 2: Inverse scattering problem

You are given that the differential cross section as calculated in Born approximation for backward scattering has the k-dependence

$$\left.\frac{d\sigma^B}{d\Omega}\right|_{backward} = A\frac{e^{-4\lambda k}}{\vec{k}^2}, \tag{3}$$

where A and λ are constants, $\lambda > 0$ and $k \equiv |\vec{k}|$.

From the above information, determine the potential $V(r)$. Note that the sign ambiguity in passing form the cross section to the scattering amplitude can be resolved by picking the plus sign!

Problem 3: Scattering by an atom

An electron scatters off a hydrogen atom in the ground state. Ignore spin and effects due to indistinguishibility of the two electrons. Assume that the potential seen by the scattering electron is

$$V(r) = -\frac{\alpha}{r} + \alpha \int d^3\vec{r}' \, \frac{\rho(r')}{|\vec{r} - \vec{r}'|} \tag{4}$$

where $\rho(\vec{r}) = |\psi_{100}(\vec{r})|^2$, the probability distribution of the bound electron.

1. What is the scattering amplitude in the Born approximation? What is the scattering length? Recall that the scattering length $a_0 = -\lim_{k\to 0} f(\theta, \phi)$.
2. In the same (Born) approximation, compute the differential and total cross sections as a function of the energy of the incident electron.

Hint: change variables in the second term to $|\vec{r} - \vec{r}'|$.