

PHY7410. Homework 6

This homework assignment is due on **March 30**.

Suggested reading:

L. Landau and E. Lifshitz, *Quantum Mechanics*, Chapter XVII (sections 132-134, 137).

Problem 1: Gauge invariance

One of the fundamental principles of modern theories of particle interactions is the principle of gauge invariance. This principle states that physical properties of a theory would not change if we perform a coordinate-dependent change of the phase of a wavefunction, while simultaneously changing (“gauging”) the vector potential. For electrodynamics it implies that physics does not change if we substitute

$$\vec{A}(r) \rightarrow \vec{A}(r) + \vec{\partial}\chi(r), \quad (1)$$

where $\chi(r)$ is an arbitrary scalar function. Let’s check that it works in quantum mechanics. To do that, consider scattering of charged particle off magnetic field $\vec{B}(r)$.

1. Using the potential written in terms of a vector potential, obtain the Born approximation scattering amplitude as a function of Fourier transform of a vector potential.
2. Prove that neither the scattering amplitude nor the cross section change under the gauge transformation of the vector potential of Eq. (1).

Problem 2: Workin’ with (pseudo)real data

Suppose that you performed a scattering experiment where for a given value of energy (supplied by your accelerator) you measured the angular distribution of scattered particles, i.e. the differential cross section as a function of an angle (see Table 1). The wave number of incident particle turned out to be $k = 3 \times 10^{12} \text{ cm}^{-1}$.

$\cos \theta$	0.9	0.7	0.5	0.3	0.1	-0.1	-0.3	-0.5	-0.7	-0.9
$d\sigma/d\Omega$ (mb/sr)	8.0	7.2	6.9	6.2	5.6	5.0	4.4	4.0	3.2	2.5

Table 1: Differential cross section from data.

1. Fit the data using

$$\frac{d\sigma}{d\Omega} = A_0 + A_1 P_1(\cos \theta) + A_2 P_2(\cos \theta), \quad (2)$$

where A_i are the constants to be determined from a least square fit.

2. Using the results from part 1, compute to two significant figures the phase shifts δ_0 and δ_1

Problem 3: Phase shifts in Born approximation

Derive an expression for the phase shift δ_l in the leading Born approximation. To achieve that, write the formula for the Born amplitude (in particular, the term $\sin(qk)/(qr)$) as an expansion over $P_l(\cos \theta)$ and then compare the obtained result to Faxen-Holtmark formula. Hint: note that

$$\frac{\sin z}{z} = J_{1/2} \left(\sqrt{\frac{\pi}{2z}} \right). \quad (3)$$