PHY8850. Homework 1

This homework assignment is due on **September 14**. The maximum attainable score for this homework, if not turned in by 5 pm that day, will be linearly decreased $N = N_{max}(1 - 0.2n)$, where n is the number of days.

Suggested reading:

M. Peskin and D. Schroeder, *"An Introduction to QFT,"* chapter 2.

Problem 1: Review of Relativistic Quantum Mechanics.

Even though a single-particle interpretation of Klein-Gordon equation (KGE) leads to internal inconsistencies, it is still instructive to work out solutions of KGE in various potentials. Find the energy spectrum and the eigenfunctions for a scalar particle in a constant magnetic field, $\vec{B} = B\vec{e}_z$. Hint: introduce magnetic field via principle of minimal substitution, $\partial_\mu \to \partial_\mu + iqA_\mu$, where A_μ is a vector potential, and look for a solution in the form (show that it is appropriate)

$$\phi(\vec{r}) = e^{-i(Et - k_x x - k_z z)}\varphi(y). \tag{1}$$

Problem 2: Dilatation symmetry.

In class, we discussed various symmetries and conserved quantities that follow from those symmetries. Let us consider one more example that could be interesting for the future studies of QCD. Consider an infinitesimal scale transformations (a.k.a. dilatations):

$$\delta x^\mu = \alpha x^\mu, \ \delta\phi(x) = D\alpha\phi(x), \tag{2}$$

where α is infinitesimly small and D is some constant. Let us see if this transformation is indeed a symmetry of a massless real scalar field theory given by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi. \tag{3}$$

(i) Show tha the action for the real scalar field $\phi(x)$ is dilatation-invariant in four space-time dimensions provided that $D = -1$

(ii) Identify the Noether current associated with this symmetry and use equations of motion to check that its divergence vanishes.

Problem 3: Equivalent Lagrangians.

In class we considered an example of a Lagrangian that led to Klein-Gordon equation of motion for a scalar field. Show that the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\phi \left(\Box + m^2 \right) \phi \tag{4}$$

also describes free motion of the scalar field, i.e describes the same “physics” as the one discussed in class. Hint: consider the action functional associated with this Lagrangian.