

PHY8850. Homework 4

This homework assignment is due on **October 25**. The maximum possible score for this homework, if not turned in by 5 pm that day, will be linearly decreased $N = N_{max}(1 - 0.2n)$, where n is the number of days.

Suggested reading:

M. Peskin and D. Schroeder, “*An Introduction to QFT*,” chapter 3-5.

Problem 1: Chiral symmetry.

Show that the action for a massless spinor field is invariant under a “chiral transformation”

$$\delta x^\mu = 0, \quad \delta \psi(x) = i\alpha \gamma_5 \psi(x), \quad (1)$$

where α is infinitesimally small. Find the Noether current associated with this transformation.

Problem 2: Fun with Dirac matrices (I).

Derive the following identities with contractions of the γ -matrices

$$\begin{aligned} \gamma_\mu \gamma_\alpha \gamma_\beta \gamma^\mu &= 4g_{\alpha\beta}, \\ \gamma_\mu \gamma_\alpha \gamma_\beta \gamma_\delta \gamma^\mu &= -2\gamma_\delta \gamma_\beta \gamma_\alpha, \\ \sigma_{\alpha\beta} \gamma_\mu \sigma^{\alpha\beta} &= 0, \\ \sigma_{\alpha\beta} \sigma^{\mu\nu} \sigma^{\alpha\beta} &= -4\sigma^{\mu\nu}. \end{aligned}$$

Hint: you don't need to use the explicit form of the γ -matrices to derive these identities.

Problem 3: Fun with Dirac matrices (II).

Expand the following products of γ -matrices in terms of the set of basis γ -matrices

$$\begin{aligned} \gamma_\mu \gamma_\nu \gamma_\rho &= ? \\ \gamma_5 \gamma_\mu \gamma_\nu &= ? \\ \sigma_{\mu\nu} \gamma_\rho \gamma_5 &= ? \\ \{\gamma^\mu, \sigma^{\nu\rho}\} &= ? \end{aligned}$$

where $\{A, B\} = AB + BA$ denotes the anticommutator.