

PHY8850. Homework 5

This homework assignment is due on **November 10**. The maximum possible score for this homework, if not turned in by 5 pm that day, will be linearly decreased $N = N_{max}(1 - 0.2n)$, where n is the number of days.

Suggested reading:

M. Peskin and D. Schroeder, “*An Introduction to QFT*,” chapter 3-5.

Problem 1: Gordon identities.

Prove the Gordon identities

$$\begin{aligned} 2m\bar{u}(\vec{p}_1)\gamma_\mu u(\vec{p}_2) &= \bar{u}(\vec{p}_1) [(p_1 + p_2)_\mu + i\sigma_{\mu\nu}(p_1 - p_2)^\nu] u(\vec{p}_2) \\ 2m\bar{v}(\vec{p}_1)\gamma_\mu v(\vec{p}_2) &= -\bar{v}(\vec{p}_1) [(p_1 + p_2)_\mu + i\sigma_{\mu\nu}(p_1 - p_2)^\nu] v(\vec{p}_2), \end{aligned} \quad (1)$$

where u and v are the spinors. In your derivation do not use any particular representation of Dirac spinors. Hint: use equations of motion.

Problem 2: Fun with quantized Dirac field.

(a) Show that quantized Dirac field satisfied the following equation

$$i[H, \psi(x)] = \frac{\partial}{\partial t}\psi(x), \quad (2)$$

where H is a Hamilton operator. Note that it is easiest to work out the commutator by expressing H and $\psi(x)$ in terms of creation and annihilation operators and using the anticommutation relations.

(b) If the Dirac field is quantized according to the Bose-Einstein statistics, what would be energy of the field (derive it)? Why is the obtained result no appropriate?

Problem 3: More fun with quantized Dirac field.

Prove that $\langle 0|T(\bar{\psi}(x)\Gamma\psi(y))|0\rangle$ is equal to zero for $\Gamma = \gamma_5$, or $\gamma_5\gamma_\mu$, but is equal to $-4img_{\mu\nu}D_F(y-x)$ for $\Gamma = \gamma_\mu\gamma_\nu$. Here $S_F(y-x)$ is a propagator

for a Dirac field, and $iS_F(x-y) = i(i \not{\partial} + m)D_F(x-y)$ with

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2 - m^2 + i\epsilon} \quad (3)$$

Hint: prove that $\langle 0|T(\bar{\psi}_a(x)\psi_b(y))|0\rangle$ is related to Feynman propagator of a Dirac field. Then rewrite $\langle 0|T(\bar{\psi}(x)\Gamma\psi(y))|0\rangle$ as a trace.