Lecture 10

- Electrodynamics
  - Direct current circuits
  - parallel and series connections
  - Kirchhoff’s rules

http://www.physics.wayne.edu/~apetrov/PHY2140/

Chapter 18
Department of Physics and Astronomy announces the Fall 2003 opening of

**The Physics Resource Center**

on Monday, September 22 in

**Room 172 of Physics Research Building.**

**Hours of operation:**

- Mondays, Tuesdays, Wednesdays: 11 AM to 6 PM
- Thursdays and Fridays: 11 AM to 3 PM

Undergraduate students taking PHY2130-2140 will be able to get assistance in this Center with their homework, labwork and other issues related to their physics course.

The Center will be open: Monday, September 22 to Wednesday, December 10, 2003.
Lightning Review

Last lecture:

1. Current and resistance

   ✓ Temperature dependence of resistance

   ✓ Power in electric circuits

\[
I = \frac{\Delta Q}{\Delta t}
\]

\[
R = R_0 \left[1 + \alpha (T - T_o)\right]
\]

\[
P = I \Delta V = I^2 R = \frac{\left(\Delta V\right)^2}{R}
\]

**Review Problem:** Consider a moose standing under the tree during the lightning storm. Is he ever in danger? What could happen if lightning hits the tree under which he is standing?
Introduction: elements of electrical circuits

- **A branch**: A branch is a single electrical element or device (resistor, etc.).

- **A junction**: A junction (or node) is a connection point between two or more branches.

If we start at any point in a circuit (node), proceed through connected electric devices back to the point (node) from which we started, without crossing a node more than one time, we form a closed-path (or loop).
18.1 Sources of EMF

- **Steady current** (constant in magnitude and direction)
  - requires a complete circuit
  - path cannot be *only* resistance
  - cannot be only potential drops in direction of current flow
- Electromotive Force (EMF)
  - provides *increase* in potential
  - converts some external form of energy into electrical energy
- Single emf and a single resistor: emf can be thought of as a “charge pump” $V = IR$

\[
V = IR = I
\]
Each real battery has some \textit{internal resistance}.

AB: potential increases by $I$ on the source of EMF, then decreases by $Ir$ (because of the internal resistance).

Thus, terminal voltage on the battery $\Delta V$ is

$$\Delta V = I - Ir$$

Note: $I$ is the same as the terminal voltage when the current is zero (open circuit).
EMF (continued)

Now add a load resistance $R$

Since it is connected by a conducting wire to the battery → terminal voltage is the same as the potential difference across the load resistance

$$\Delta V = I - Ir = IR, \quad \text{or}$$

$$I = Ir + IR$$

Thus, the current in the circuit is

$$I = \frac{I}{R + r}$$

Power output:

$$II = I^2r + I^2R$$

Note: we’ll assume $r$ negligible unless otherwise is stated
Measurements in electrical circuits

Voltmeters measure Potential Difference (or voltage) across a device by being placed in parallel with the device.

Ammeters measure current through a device by being placed in series with the device.
Direct Current Circuits

Two Basic Principles:
- Conservation of Charge
- Conservation of Energy

Resistance Networks

\[ V_{ab} = IR_{eq} \]
\[ R_{eq} \equiv \frac{V_{ab}}{I} \]
18.2 Resistors in series

1. Because of the charge conservation, all charges going through the resistor $R_2$ will also go through resistor $R_1$. Thus, currents in $R_1$ and $R_2$ are the same,

$$I_1 = I_2 = I$$

2. Because of the energy conservation, total potential drop (between A and C) equals to the sum of potential drops between A and B and B and C,

$$\Delta V = IR_1 + IR_2$$

By definition,

$$\Delta V = IR_{eq}$$

Thus, $R_{eq}$ would be

$$R_{eq} \equiv \frac{\Delta V}{I} = \frac{IR_1 + IR_2}{I} = R_1 + R_2$$

$$R_{eq} = R_1 + R_2$$
Resistors in series: notes

Analogous formula is true for any number of resistors,

\[ R_{eq} = R_1 + R_2 + R_3 + \ldots \]  

(series combination)

It follows that the equivalent resistance of a series combination of resistors is greater than any of the individual resistors.
Resistors in series: example

In the electrical circuit below, find voltage across the resistor $R_1$ in terms of the resistances $R_1$, $R_2$ and potential difference between the battery’s terminals $V$.

Energy conservation implies:

$$V = V_1 + V_2$$

with $V_1 = IR_1$ and $V_2 = IR_2$

Then,

$$V = I(R_1 + R_2)$$

so $I = \frac{V}{R_1 + R_2}$

Thus,

$$V_1 = V \frac{R_1}{R_1 + R_2}$$

This circuit is known as **voltage divider**.
18.3 Resistors in parallel

1. Since both $R_1$ and $R_2$ are connected to the same battery, potential differences across $R_1$ and $R_2$ are the same,

$$V_1 = V_2 = V$$

2. Because of the charge conservation, current, entering the junction A, must equal the current leaving this junction,

$$I = I_1 + I_2$$

By definition,

$$I = \frac{V}{R_{eq}}$$

Thus, $R_{eq}$ would be

$$I = \frac{V}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{V}{R_1} + \frac{V}{R_2}$$

or

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$
Resistors in parallel: notes

- Analogous formula is true for any number of resistors,
  \[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]  
  (parallel combination)

- It follows that the equivalent resistance of a parallel combination of resistors is always less than any of the individual resistors
Resistors in parallel: example

In the electrical circuit below, find current through the resistor $R_1$ in terms of the resistances $R_1$, $R_2$ and total current $I$ induced by the battery.

Charge conservation implies:

$$I = I_1 + I_2$$

with

$$I_1 = \frac{V}{R_1}, \text{ and } I_2 = \frac{V}{R_2}$$

Then,

$$I_1 = \frac{IR_{eq}}{R_1}, \text{ with } R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

Thus,

$$I_1 = I \frac{R_2}{R_1 + R_2}$$

This circuit is known as current divider.
Direct current circuits: example

Find the currents $I_1$ and $I_2$ and the voltage $V_x$ in the circuit shown below.

Strategy:

1. Find current $I$ by finding the equivalent resistance of the circuit
2. Use current divider rule to find the currents $I_1$ and $I_2$
3. Knowing $I_2$, find $V_x$. 

![Circuit Diagram](image)
Direct current circuits: example

Find the currents $I_1$ and $I_2$ and the voltage $V_x$ in the circuit shown below.

First find the equivalent resistance seen by the 20 V source:

$$R_{eq} = 7\Omega + \frac{4\Omega(12\Omega)}{12\Omega + 4\Omega} = 10 \Omega$$

Then find current $I$ by,

$$I = \frac{20V}{R_{eq}} = \frac{20V}{10\Omega} = 2 \, A$$

We now find $I_1$ and $I_2$ directly from the current division rule:

$$I_1 = \frac{2A(4\Omega)}{12\Omega + 4\Omega} = 0.5 \, A, \text{ and } I_2 = I - I_1 = 1.5 \, A$$

Finally, voltage $V_x$ is

$$V_x = I_2 (4\Omega) = 1.5 \, A (4\Omega) = 6V$$
18.4 Kirchhoff’s rules and DC currents

The procedure for analyzing complex circuits is based on the principles of conservation of charge and energy. They are formulated in terms of two Kirchhoff’s rules:

1. The sum of currents entering any junction must equal the sum of the currents leaving that junction (current or junction rule).

2. The sum of the potential differences across all the elements around any closed-circuit loop must be zero (voltage or loop rule).
a. Junction rule

As a consequence of the Law of the conservation of charge, we have:

- The sum of the currents entering a node (junction point) equal to the sum of the currents leaving.

\[ I_a + I_b = I_c + I_d \]

Similar to the water flow in a pipe.

\( I_a, I_b, I_c, \) and \( I_d \) can each be either a positive or negative number.
b. Loop rule

As a consequence of the Law of the conservation of energy, we have:

- The sum of the potential differences across all the elements around any closed loop must be zero.

1. Assign symbols and directions of currents in the loop
   - If the direction is chosen wrong, the current will come out with a right magnitude, but a negative sign (it’s ok).

2. Choose a direction (cw or ccw) for going around the loop.
   Record drops and rises of voltage according to this:
   - If a resistor is traversed in the direction of the current: \(+V = +IR\)
   - If a resistor is traversed in the direction opposite to the current: \(-V=-IR\)
   - If EMF is traversed “from – to + ”: +I
   - If EMF is traversed “from + to – ”: -I
Loops can be chosen arbitrarily. For example, the circuit below contains a number of closed paths. Three have been selected for discussion.

Suppose that for each element, respective current flows from + to - signs.
b. Loop rule: illustration

Using sum of the drops = 0

Blue path, starting at “a”

- \( v_7 + v_{10} - v_9 + v_8 = 0 \)

Red path, starting at “b”

+\( v_2 - v_5 - v_6 - v_8 + v_9 - v_{11} \)
- \( v_{12} + v_1 = 0 \)

Yellow path, starting at “b”

+\( v_2 - v_5 - v_6 - v_7 + v_{10} - v_{11} \)
- \( v_{12} + v_1 = 0 \)
Kirchhoff’s Rules: Single-loop circuits

Example: For the circuit below find $I$, $V_1$, $V_2$, $V_3$, $V_4$ and the power supplied by the 10 volt source.

1. For convenience, we start at point “a” and sum voltage drops =0 in the direction of the current $I$.

\[
+10 - V_1 - 30 - V_3 + V_4 - 20 + V_2 = 0 \quad (1)
\]

We note that: $V_1 = -20I$, $V_2 = 40I$, $V_3 = -15I$, $V_4 = 5I$ \quad (2)

We substitute the above into Eq. 1 to obtain Eq. 3 below.

\[
10 + 20I - 30 + 15I + 5I - 20 + 40I = 0 \quad (3)
\]

Solving this equation gives, $I = 0.5$ A.
Kirchhoff’s Rules: Single-loop circuits (cont.)

Using this value of I in Eq. 2 gives:

\[ V_1 = -10 \text{ V} \quad V_3 = -7.5 \text{ V} \]
\[ V_2 = 20 \text{ V} \quad V_4 = 2.5 \text{ V} \]
\[ P_{10\text{(supplied)}} = -10I = -5 \text{ W} \]

(We use the minus sign in \(-10I\) because the current is entering the + terminal)
In this case, power is being absorbed by the 10 volt supply.
18.5 RC circuits

Consider the circuit

\[ v_c = \frac{q}{C} \quad v_R = iR \]

\[ q = Q \left(1 - e^{-t/RC}\right) \]

RC is called the time constant
**RC circuits**

Discharge the capacitor

\[ q = Qe^{-t/RC} \]

\[ v_c = \frac{q}{C} \quad v_R = iR \]