PHY8860. Homework 6

This homework assignment is due on November 5. The maximum possible score of this homework, if not turned in by 5 pm that day, will be linearly decreased $N = N_{\text{max}}(1 - 0.2n)$, where $n$ is the number of days.

Suggested reading:
M. Peskin and D. Schroeder, “An Introduction to Quantum Field Theory” chapters 8-12.

Problem 1: Scalar QED

Consider the theory of a complex scalar field $\phi$ interacting with the electromagnetic field $A_\mu$,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* (D^\mu \phi) - m^2 \phi^* \phi, \quad (1)$$

where $D_\mu = \partial_\mu + ieA_\mu$ is the gauge-covariant derivative.

(a) use functional methods to derive the propagator of the complex scalar field. Derive the rest of the Feynman rules. Please show your work.

(b) calculate the total cross section $\sigma$ of $e^+e^- \to \phi^*\phi$ process (use usual QED Feynman rules for the $e^+e^-\gamma$ vertex). Find the asymptotic limit of $\sigma(E \to \infty)$ and compare it to the $e^+e^- \to \mu^+\mu^-$ cross section.

Problem 2: Sigma model

The Lagrangian density of the sigma model is given by

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \pi)^2 - \frac{m^2}{2} \sigma^2 - \lambda v \sigma^3 - \lambda v \sigma \pi^2 - \frac{\lambda}{4} \left( \sigma^2 + \pi^2 \right)^2, \quad (2)$$

where $\sigma$ and $\pi$ are scalar fields, and $v^2 = m^2/(2\lambda)$. As you can see, classically, the field $\pi$ is massless. Show that it also remains massless when the one-loop corrections are included. To achieve this,

(a) derive Feynman rules for all the vertices given by the Lagrangian in Eq. (2). To distinguish different fields on your graph, use solid line to denote $\pi$ field and dashed line to denote $\sigma$ field.
(b) draw all (five) one loop self-energy diagrams that contribute to $\delta m_{\pi}^2 = \Sigma(p^2)$ (don’t forget the diagrams with tadpole insertions or explicitly argue and show how to eliminate them).

(c) calculate all diagrams using the Feynman rules derived in (a) to derive total contribution to the self-energy of $\pi$. Be careful about symmetry factors! At the end, show that $m_{\pi}^2 = 0 + \Sigma(0) = 0$. 

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