PHY8860. Homework 8 (the last one)

This homework assignment is due on November 30. The maximum possible score of this homework, if not turned in by 5 pm that day, will be linearly decreased $N = N_{\text{max}}(1 - 0.2n)$, where $n$ is the number of days.

Suggested reading:

M. Peskin and D. Schroeder, “An Introduction to Quantum Field Theory” chapters 8-12.

Problem 1: Triangle diagram.

In regular QED (and using dimensional regularization), find the divergent part of the diagram

![Triangle Diagram]

Prove that this diagram cancels with the diagram of the reverse orientation inside the fermion loop (Furry’s theorem).

Problem 2: The $\rho$-meson decay constant.

Consider the decay $\rho^0(p, \lambda) \rightarrow l^+(p_+, \lambda_+) + l^-(p_-, \lambda_-)$, where $\rho$-meson is a spin-one state with $m_\rho \approx 770 \text{ MeV}$. It is a second order electromagnetic transition, induced by

$$\mathcal{L}_\text{em} = -eA_\mu (J^\mu_{\text{em}}(\text{quark}) + J^\mu_{\text{em}}(\text{lept})).$$  \hspace{1cm} (1)
Since this decay involves electromagnetic annihilation of light quarks bound into the $\rho$-meson, we have to know their wave function or interaction potential in order to solve this problem. The simplifying argument in treating this decay is that those nonperturbative effects can be parametrized by introducing the $\rho$-meson decay constant $g_\rho$,

$$\langle 0 | J^m_\mu (\text{quark}) | \rho^0 (p, \lambda) \rangle = g_\rho \epsilon_\mu (p, \lambda).$$

(a) Find the decay width $\Gamma_{\rho \rightarrow l^+ l^-}$ as a function of $g_\rho$ and $x_l = m_l^2 / m_\rho^2$, where $m_l$ is mass of one of the final state leptons.

(b) Determine $g_\rho$ numerically using the measured width for $\rho \rightarrow e^+ e^-$ from “Review of Particle Physics.”