Final Exam

Multiple Choice (10 points each) Identify the letter of the choice that best completes the statement or answers the question.

1. A ping-pong ball covered with a conducting graphite coating has a mass of $5.0 \times 10^{-3}$ kg and a charge of 4.0 $\mu$C. What electric field directed upward will exactly balance the weight of the ball? ($g = 9.8$ m/s$^2$)
   a. $8.2 \times 10^2$ N/C
   b. $1.2 \times 10^4$ N/C
   c. $2.0 \times 10^3$ N/C
   d. $5.1 \times 10^6$ N/C
   e. $3.4 \times 10^{-3}$ N/C
   
   
   
   

2. Two capacitors with capacitances of 1.5 and 0.25 $\mu$F, respectively, are connected in parallel. The system is connected to a 50-V battery. What charge accumulates on the 1.5-$\mu$F capacitor?
   a. 100 $\mu$C
   b. 75 $\mu$C
   c. 50 $\mu$C
   d. 33 $\mu$C
   e. 25 $\mu$C

3. If a metallic wire of cross sectional area $3.0 \times 10^{-6}$ m$^2$ carries a current of 6.0 A and has a mobile charge density of $4.24 \times 10^{28}$ carriers/m$^3$, what is the average drift velocity of the mobile charge carriers? (charge value = $1.6 \times 10^{-19}$ C)
   a. $3.4 \times 10^3$ m/s
   b. $1.7 \times 10^3$ m/s
   c. $1.5 \times 10^4$ m/s
   d. $2.9 \times 10^4$ m/s
   e. $1.2 \times 10^{-1}$ m/s

   
   
   
   

\[ I = n g \nu_d A \]
\[ A = 3 \times 10^{-6} \text{ m}^2 \]
\[ I = 6 \text{ A} \]
\[ n = 4.24 \times 10^{28} \text{ /m}^3 \]
\[ \nu_d = \frac{I}{n g A} \]
\[ \frac{6 \text{ C/s}}{1.6 \times 10^{-19} \text{ C} \cdot 4.24 \times 10^{28} \text{ /m}^3 \cdot 3 \times 10^{-6} \text{ m}^2} \]
\[ 2.9 \times 10^{-4} \text{ m/s} \]
4. If $E = 20 \text{ V}$, at what rate is thermal energy being generated in the 20-$\Omega$ resistor?

First find total current, 

$\Rightarrow$ Find equivalent resistance.

\[ 3 \text{ } 30 \Omega \text{ in parallel } = 10 \Omega \]

Then in series \[ 40 \Omega + 10 \Omega + 20 \Omega = 70 \Omega \]

So \[ I = \sqrt{\frac{E}{R_{eq}}} = \frac{20\text{ V}}{70 \Omega} = 0.286 \text{ A} \]

Then \[ \text{Power dissipated } = I^2 R = (0.286)^2 \times 20 \text{ W } = \boxed{1.6 \text{ W}} \]

5. A 100-m-long wire carrying a current of 4.0 A will be accompanied by a magnetic field of what strength at a distance of 0.050 m from the wire? (magnetic permeability in empty space $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$)

\[ B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 4}{2\pi} \frac{4}{0.050 \text{ m}} \text{ Tesla} = \boxed{1.6 \times 10^{-5} \text{ T}} \]

6. A circuit consists of a 10-mH coil, a 12-$\Omega$ resistor, a 6.0-$\Omega$ resistor, a 9.0-V battery and a switch, all in series. What is the time constant of this circuit?

\[ \tau = \frac{L}{R} \]

\[ R = R_1 + R_2 = 18 \Omega \]

\[ \therefore \tau = \frac{10 \times 10^{-3} \text{ H}}{18 \Omega} = \boxed{5.6 \times 10^{-4} \text{ sec.}} \]
7. A radio wave transmits 1.2 $\text{W/m}^2$ average power per unit area. What is the peak value of the associated magnetic field? ($\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}$ and $c = 3.00 \times 10^8 \text{m/s}$)

\[ P = \frac{c \, B_{\text{max}}^2}{2 \mu_0} \Rightarrow B_{\text{max}} = \sqrt{\frac{2 \mu_0 \, P}{c}} \]

\[ P = 1.2 \text{ W/m}^2, \text{ solving yields} \]

\[ B_{\text{max}} = 1.0 \times 10^{-7} \text{T} \]

- a. $1.0 \times 10^{-7} \text{T}$
- b. $8.4 \times 10^{-3} \text{T}$
- c. $1.2 \text{T}$
- d. $30 \text{T}$
- e. $51 \text{T}$

8. A rocket ship is 80.0 m in length when measured before leaving the launching pad. What would its velocity be if a ground observer measured its length as 60.0 m while it is in flight? ($c = 3.00 \times 10^8 \text{m/s}$)

\[ L_p = 80.0 \text{ m} \quad L_{\text{flight}} = 60.0 \text{ m} \]

\[ \gamma = \frac{L_p}{L_{\text{flight}}} \]

\[ \gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}} \quad \frac{c}{v} = \sqrt{1 - \frac{v^2}{c^2}} = 0.66 \Rightarrow v = 1.98 \times 10^8 \text{ m/s} \]

- a. $0.980 \times 10^8 \text{ m/s}$
- b. $1.15 \times 10^8 \text{ m/s}$
- c. $1.33 \times 10^8 \text{ m/s}$
- d. $1.98 \times 10^8 \text{ m/s}$
- e. $2.55 \times 10^8 \text{ m/s}$

9. Starting from rest, an electron accelerates through a potential difference of 40 V. What is its de Broglie wavelength? ($h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}, m_e = 9.11 \times 10^{-31} \text{ kg}, \text{ and } 1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$)

\[ \lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{Need to find } v. \]

\[ \frac{1}{2} mv^2 = eV = e \times 40 \text{ V} \Rightarrow v = \sqrt{\frac{2 \times 40 \text{ eV}}{m_e}} = 3.75 \times 10^6 \text{ m/s} \]

\[ \lambda = \frac{h}{m_e v} = 1.9 \times 10^{-10} \text{ m} \]

- a. $1.1 \times 10^{-10} \text{ m}$
- b. $1.5 \times 10^{-10} \text{ m}$
- c. $1.9 \times 10^{-10} \text{ m}$
- d. $2.3 \times 10^{-10} \text{ m}$
- e. $3.6 \times 10^{-10} \text{ m}$

10. Tritium is radioactive with a half-life of 12.33 years decaying into $^3\text{He}$ with low-energy electron emission. If we have a sample of $3.00 \times 10^{15}$ tritium atoms, what is its activity in decays/second? (1 year = $3.15 \times 10^7$ s)

\[ R = \text{activity} = \frac{\Delta N}{\Delta t} = \lambda N \quad \text{but} \quad \lambda = \frac{\ln 2}{T_{1/2}} \]

\[ R = \frac{N \ln 2}{T_{1/2}} \quad \text{but} \quad T_{1/2} = 12.33 \text{ years} \]

\[ R = 5.35 \times 10^9 \text{ sec}^{-1} \]

- a. $4.20 \times 10^{10} \text{/second}$
- b. $5.35 \times 10^9 \text{/second}$
- c. $3.69 \times 10^7 \text{/second}$
- d. $6.64 \times 10^7 \text{/second}$
- e. $7.72 \times 10^6 \text{/second}$
11. A hydrogen atom in the ground state absorbs a 12.75 eV photon. To what level is the electron promoted? (The ionization energy of hydrogen is 13.6 eV.)
   a. \( n = 2 \)
   b. \( n = 3 \)
   c. \( n = 4 \)
   d. \( n = 5 \)
   e. \( n = 6 \)

   \[ E_n = \frac{-13.6 \text{ eV}}{n^2} = -0.85 \text{ eV} \quad \Rightarrow \quad n^2 = \frac{13.6}{0.85} = 16 \]
   \[ \Rightarrow \quad n = 4 \]

12. Calculate the energy released in the following fusion reaction where reactants are \(^6\text{Li}\) and a neutron; products are \(^4\text{He}\) and \(^1\text{H}\). (atomic masses: \(^6\text{Li}, 6.01512\); neutron, 1.00867; \(^4\text{He}, 4.00260\); \(^1\text{H}, 3.016031\); also 1 u = 931.5 MeV/c^2)
   a. 2.95 MeV
   b. 4.81 MeV
   c. 8.63 MeV
   d. 17.2 MeV
   e. 34.5 MeV

   \[ \Delta m = 0.005159 \text{ u} \quad E = mc^2 = \Delta m \cdot 931.5 \text{ MeV/c}^2 \]
   \[ E = 4.81 \text{ MeV} \]

Short Answer (15 points)
Show all your work in solving this problem.

13. If a fossil bone is found to contain \( \frac{1}{16} \)th as much Carbon-14 as the bone of a living animal, what is the approximate age of the fossil? (half-life of \(^{14}\text{C}\) = 5730 years)

   \[ \frac{N}{N_0} = \frac{1}{16} = e^{-\lambda t} \quad \Rightarrow \quad 16 = e^{\lambda t} \quad \Rightarrow \quad \ln 16 = \lambda t \]
   \[ \Rightarrow \quad t = \frac{\ln 16}{\lambda} = \frac{\ln 16}{\ln 2}, T_{1/2} = 4 \cdot T_{1/2} \]

   \[ \lambda = \frac{\ln 2}{T_{1/2}} \]

   \[ \text{Age} = 4 \cdot 5730 \text{ years} = 22920 \approx 22900 \text{ years} \]